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Theoretical modelling and experimental study of the fatigue of elastomers under cyclic loadings of variable amplitude

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A B S T R A C T

Deviations from Miner's linear law of cumulative damage have been modelled and observed many times for the fatigue of metals, but almost no analogous studies have been performed for elastomers. Such a study is reported here.

A simple phenomenological model, applicable to any type of material and able to quantitatively reproduce such deviations, is presented first. This model is based on continuum damage mechanics. It relates the fatigue damage of the material to the number of cycles through some suitable evolution law, in which the derivative of damage is expressed as a non-factorizable function of the instantaneous load cycle and the damage itself.

Fatigue experiments performed on "diabolo" specimens made of two different elastomeric materials and subjected to two successive cyclic loads of different amplitudes are then reported. Significant deviations from Miner's rule are observed: Miner's "total cumulated damage" may be lower or larger than unity by a small or large amount, depending on the sequence of loadings and the type of material. As a rule, the deviation from Miner's rule systematically changes sign upon reversal of the sequence of loadings. The model is shown to allow an acceptable reproduction of the experimental results, and especially of this systematic change of sign.

1. Introduction

Rubber components often experience complex cyclic loadings involving 3D stress states varying arbitrarily in time. Prediction of fatigue under such general conditions is important for the prediction of the durability of these components.

Models for the fatigue of elastomers under general cyclic loadings have been reviewed by Mars and Fatemi [1], who distinguished between models based on consideration of crack initiation and propagation, respectively. We shall focus here on the first class of models, initiated by Cadwell et al. [2], since propagation-based models are applicable only when the location and size of the initial crack(s) are known, which is seldom the case in practice.

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In a previous paper (Brunac et al. [3]), we considered the problem of predicting the fatigue lifetime under general 3D but perfectly time-periodic cyclic loadings. We now envisage the question of description of fatigue under cyclic loadings varying in time.

The standard answer to this problem consists of using Miner's [4] heuristic, but appealingly simple and elegant linear rule of cumulative damage. The applicability and limits of this rule have been assessed in numerous experimental works in the case of metals; as a rule, in such materials, Miner's "total cumulated damage" is observed to be smaller than unity if the more severe load cycles are applied first, and larger than unity if the less severe cycles are applied first (Chaboche and Lesne [5]). In contrast, very few similar studies have been performed for elastomers. The purpose of this paper is therefore to describe a theoretical and experimental investigation of deviations from Miner's rule for the fatigue of elastomers.

The paper is organized as follows:

- Section 2 first describes a heuristic model retaining the basic simplicity of Miner's rule, but accounting for possible deviations from its predictions while allowing for an arbitrary dependence of the number of cycles at failure upon the load cycle. This model belongs to the category of continuum damage models, as described in the book of Lemaître and Chaboche [6], and relies on some evolution equation for the damage variable in which this variable and the instantaneous load cycle[1] appear in a "non-factorized" form. It involves a single adjustable parameter depending on the load cycle.
- Section 3 then explains how the load-cycle-dependent material parameter introduced in the model may be determined from experimental numbers of cycles at failure observed in successions of different cyclic loads.
- Section 4 presents the set of experiments performed. These experiments involve load histories consisting of two cyclic loads of different amplitudes applied in succession on "diabolo" specimens made of two distinct elastomeric materials. They evidence clear and significant deviations from Miner's rule in at least one of these materials.
- Finally Section 5 presents the application of the model to the experiments performed. Determination of the values of the model parameter for the two load cycles considered in these experiments is shown to allow a much better reproduction of the experimental results than Miner's standard rule.

2. Continuum-damage-based description of fatigue under varying cyclic loadings

A number of models accounting for possible deviations from Miner's rule of cumulative damage have been proposed. An extensive overview of the literature was provided by Fatemi and Yang [7]; historical references include the works of Marko and Starkey [8], Manson [9], Bui-Quoc et al. [10] and Subramanyan [11], among others. We shall focus here on models based on continuum damage mechanics.

2.1. Generalities

The application of the phenomenological theory of continuum damage mechanics to the description of fatigue was explained in the book of Lemaître and Chaboche [6], and a more recent review was provided by Desmorat [12]. In this approach, the initiation of a macroscopic fatigue crack in an elementary volume of material is considered to be due to the progressive degradation of this volume, described through some damage variable D lying in the interval $(0, 1)$. This variable reduces the specific free energy by the factor $1 - D$ or some variant.[2] It obeys an evolution equation in which the number of cycles N plays the role of time. The simplest possible one reads

$$\frac{\mathrm{d}D}{\mathrm{d}N} = \frac{1}{N_\mathrm{f}}$$

where $N_\mathrm{f} \equiv N_\mathrm{f}(\mathcal{C})$ denotes the number of cycles at failure of the elementary volume, depending on the current load cycle $\mathcal{C}$.[3] This law reproduces Miner's rule since the damage after N non-identical cycles (with different values of N_f) is

$$D = \int_0^N \frac{\mathrm{d}N}{N_\mathrm{f}}$$

A more general form of the evolution law of D is

$$\frac{\mathrm{d}D}{\mathrm{d}N} = f(D, \mathcal{C}) \tag{1}$$

for some non-negative function f. When the load cycle $\mathcal{C}$ is invariable in time, integration of this evolution law yields

$$\int_0^D \frac{\mathrm{d}D'}{f(D', \mathcal{C})} = N$$

[1] This expression refers to the *sequence of loadings* undergone by the material during the cycle; the *number* of this cycle is immaterial.

[2] A factor of $1 - D^\alpha$, with $\alpha > 1$, was argued by Brunac [13] to be more appropriate in the case of elastomers, on the grounds that the elastic stiffness is experimentally found to remain almost constant during the major part of the degradation process and decrease significantly only at its very end.

[3] That is, again, on the sequence of loadings undergone by the material during the cycle, not on the number of this cycle.