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Separation flow control

Open-loop control of a separated boundary layer

Contrôle en boucle ouverte d'une couche limite décollée

Édouard Boujo [a,*], François Gallaire [a], Uwe Ehrenstein [b]

[a] *Laboratory of Fluid Mechanics and Instabilities, École polytechnique fédérale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland*
[b] *Aix–Marseille Université, CNRS, Centrale Marseille, IRPHE UMR 7342, 13384, Marseille, France*

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ABSTRACT

Linear optimal gains $G_{\mathrm{opt}}(\omega)$ are computed for the separated boundary-layer flow past a two-dimensional bump in the subcritical regime. Very large values are found, making it possible for small-amplitude noise to be strongly amplified and to destabilize the flow. Next, a variational technique is used to compute the sensitivity of $G_{\mathrm{opt}}(\omega)$ to steady control (volume force in the flow, or blowing/suction at the wall). The bump summit is identified as the region the most sensitive to wall control. Based on these (linear) sensitivity results, a simple open-loop control strategy is designed, with steady wall suction at the bump summit. Calculations on non-linear base flows confirm that optimal gains can be significantly reduced at all frequencies using this control. Finally, sensitivity analysis is applied to the length of the recirculation region l_c and reveals that the above control configuration is also the most efficient to shorten the recirculation region. This suggests that l_c is a relevant macroscopic parameter to characterize wall-bounded separated flows, which could be used as a proxy for energy amplification when designing steady open-loop control.

RÉSUMÉ

Le gain optimal linéaire $G_{\mathrm{opt}}(\omega)$ est calculé pour un écoulement de couche limite décollée en aval d'une bosse bidimensionnelle, en régime sous-critique. De très grandes valeurs sont obtenues. Un bruit de faible amplitude peut donc être fortement amplifié et déstabiliser l'écoulement. Une technique variationnelle est utilisée pour calculer la sensibilité de $G_{\mathrm{opt}}(\omega)$ à un contrôle stationnaire (force volumique dans l'écoulement, ou soufflage/aspiration à la paroi). Le sommet de la bosse est identifié comme la région la plus sensible au contrôle pariétal. À partir de ces résultats (linéaires), une stratégie simple de contrôle en boucle ouverte est développée, avec aspiration stationnaire au sommet de la bosse. Des calculs sur des champs de base non linéaires confirment que ce contrôle réduit significativement le gain optimal à toutes les fréquences. Enfin, l'analyse de sensibilité est appliquée à la longueur de la zone de recirculation l_c et révèle que la configuration de contrôle ci-dessus est aussi la plus efficace pour raccourcir la zone de recirculation. Cela suggère que l_c est un paramètre macroscopique pertinent pour caractériser les écoulements

* Corresponding author.
E-mail addresses: edouard.boujo@epfl.ch (É. Boujo), francois.gallaire@epfl.ch (F. Gallaire), ehrenstein@irphe.univ-mrs.fr (U. Ehrenstein).

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décollés près d'une paroi, qui pourrait être utilisé comme alternative à l'amplification d'énergie lors de l'élaboration d'un contrôle stationnaire en boucle ouverte.

1. Introduction

Some flows undergo transition below the critical Reynolds number Re_c predicted by linear stability analysis, e.g. parallel flows such as Couette and Hagen–Poiseuille (linearly stable for all Reynolds numbers [1]) and non-parallel configurations such as jets or the flow past a backward-facing step. In these flows, classical linear stability theory, which focuses on the long-term fate of small perturbations, predicts that all linear eigenmodes are damped for $\mathrm{Re} < \mathrm{Re}_c$, but it has become clear in the past decades that perturbations can be amplified by non-modal mechanisms [2]. While eigenvalues are not relevant in this context, non-modal mechanisms are well characterized by two complementary ideas: transient growth of initial conditions, and asymptotic amplification of forcing. These mechanisms are a result of the non-normality of the linearized Navier–Stokes operator which governs the dynamics of perturbations. For example, non-normality leads to large transient growth in parallel shear flows through the two-dimensional (2D) Orr mechanism and, more importantly, the three-dimensional (3D) lift-up effect [3]; in non-parallel flows, large transient growth is observed because of convective non-normality [4]. Today, transient growth is a well-established notion, and most attempts to control convectively unstable flows naturally focus on reducing the largest possible transient growth, or "optimal growth" [5], but recently optimal response to harmonic forcing, or "optimal gain", has drawn increasing attention too [6–8]. Brandt et al. [9] introduced a method to quantify the sensitivity of the largest asymptotic amplification to steady control, and applied it to a flat plate boundary layer. In this study, the flow past a wall-mounted bump is considered (Section 2). This separated flow is characterized by a long recirculation region, high shear, strong backflow, and exhibits large transient growth [10,11]. Optimal gains are computed at different frequencies (Section 3), and a sensitivity analysis is performed in order to identify regions where these gains can be reduced with steady open-loop control (Section 4.1). Sensitivity analysis is also applied to the length of the recirculation region (Section 4.2). Comparing the two analyses suggests that the recirculation length could be used as a single characteristic parameter when designing steady open-loop control for separated wall-bounded flows.

2. Problem description

The flow past a 2D bump mounted on a flat plate is considered. The bump geometry $y = y_b(x)$ is shown in Fig. 1 and is the same as in Marquillie and Ehrenstein [12] and following studies [10,11,13]. The incoming flow has a Blasius boundary layer profile, of displacement thickness δ^* at the reference position $x = 0$. The bump summit is located at $x = x_b = 25\delta^*$, and the bump height is $h = 2\delta^*$. All quantities in the problem are made dimensionless with inlet velocity U_∞ and inlet boundary layer displacement thickness δ^*. The Reynolds number is defined as $\mathrm{Re} = U_\infty \delta^*/\nu$, with ν the fluid kinematic viscosity. Previous studies using direct numerical simulations [12] and linear global stability analysis [10] reported a 2D critical Reynolds number Re_c between 590 and 610. (See [14] for details about the 3D flow.) In this study, we focus on the 2D flow at Reynolds number $\mathrm{Re} \le 580$.

The fluid motion in the domain Ω is described by the state vector $\mathbf{Q} = (\mathbf{U}, P)^T$ (velocity and pressure fields), solution of the 2D incompressible Navier–Stokes equations:

$$\begin{aligned} &\nabla \cdot \mathbf{U} = 0, \qquad \partial_t \mathbf{U} + \nabla\mathbf{U} \cdot \mathbf{U} + \nabla P - \mathrm{Re}^{-1}\nabla^2\mathbf{U} = \mathbf{F} + \mathbf{C} \quad \text{in } \Omega \\ &\mathbf{U} = \mathbf{U}_c \quad \text{on } \Gamma_w \end{aligned} \tag{1}$$

Here, $\mathbf{F}(t)$ is a time-dependent volume force, aiming to model external (uncontrolled) noise. In order to alter the flow and modify its properties (e.g. reduce noise amplification), steady control can be applied: volume control $\mathbf{C}$, or blowing/suction $\mathbf{U}_c$ at the wall. In the absence of external forcing, the steady-state base flow $\mathbf{Q}_b = (\mathbf{U}_b, P_b)^T$ is solution of:

$$\begin{aligned} &\nabla \cdot \mathbf{U}_b = 0, \qquad \nabla\mathbf{U}_b \cdot \mathbf{U}_b + \nabla P_b - \mathrm{Re}^{-1}\nabla^2\mathbf{U}_b = \mathbf{C} \quad \text{in } \Omega \\ &\mathbf{U}_b = \mathbf{U}_c \quad \text{on } \Gamma_w \end{aligned} \tag{2}$$

All steady-state base flows $\mathbf{Q}_b$, with or without control, are computed with an iterative Newton method, convergence being reached when the residual is smaller than 10^{-12} in L^2 norm. A 2D triangulation of the computational domain Ω ($0 \le x \le 400$, $y_b \le y \le 50$) is generated with the finite element software *FreeFem++* (http://www.freefem.org), and Eqs. (2) are solved in their variational formulation, with the following boundary conditions: Blasius profile $\mathbf{U}_b = (U_{\mathrm{Blasius}}, 0)^T$ at the inlet, blowing/suction $\mathbf{U}_b = \mathbf{U}_c$ or no-slip condition $\mathbf{U}_b = \mathbf{0}$ at the wall, symmetry condition $\partial_y U_b = V_b = 0$ at the top border, and outflow condition $-P_b\mathbf{n} + \mathrm{Re}^{-1}\nabla\mathbf{U}_b \cdot \mathbf{n} = \mathbf{0}$ at the outlet, with $\mathbf{n}$ the outward unit normal vector. P2 and P1 Taylor–Hood elements are used for spatial discretization of velocity and pressure, respectively.