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New analytical solutions for weakly compressible Newtonian Poiseuille flows with pressure-dependent viscosity

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ABSTRACT

Steady-state, isothermal, Poiseuille flows in straight channels and circular tubes of weakly compressible Newtonian fluids are considered. The major assumption is that both the mass density and the shear viscosity of the fluid vary linearly with pressure. The non-zero velocity components, the pressure, the mass density and viscosity of the fluid are represented over the flow domain as asymptotic expansions in which the dimensionless isothermal compressibility coefficient ε is taken as small parameter. A perturbation analysis is performed and asymptotic solutions for all variables are obtained up to first order in ε. The derived solutions, which hold for not necessarily small values of the dimensionless pressure-dependence coefficient, extend previous regular perturbation results and analytical works in the literature for weakly compressible fluids with constant viscosity (solved with a regular perturbation scheme), for incompressible flows with pressure-dependent viscosity (solved analytically), as well as for compressible fluids with pressure-dependent viscosity (solved with double regular perturbation schemes). In contrast to the previous analytical studies in the literature, a non-zero wall-normal velocity is predicted at first order in ε, even at zero Reynolds number. A severe reduction of the volumetric flow-rate at the entrance of the tube/channel and multiplicity of solutions in the flow curves (volumetric flow-rate versus pressure drop) are also predicted. Last, it is shown that weak compressibility of the fluid and the viscosity pressure-dependence have competing effects on the mean friction factor and the average pressure difference required to drive the flow.

1. Introduction

The density and the viscosity of a fluid depend on both the temperature and the pressure. In certain cases, the dependence of the viscosity on pressure may be much stronger than that of the density, e.g., with polymer melts (Denn, 2008; Renardy, 2003) and lubricants (Rajagopal, 2006; Rajagopal, Saccomandi, & Vergori, 2012). This dependence becomes important in many applications involving high pressures or a large pressure range, such as polymer and food processing (Dealy & Wang, 2013), crude oil and fuel oil pumping (Martinez-Boza, Martin-Alfonso, Callegos, & Fernández, 2011; Schaschke, Fletcher, & Glen, 2013), fluid film lubrication (Hamrock, Schmid, & Jaconson, 2004), microfluidics (Silber-Li, Cui, Tan, & Tabeling, 2006), filtration through porous media (Fusi, Farina, & Rosso, 2015), certain geophysical flows (Schoof, 2007; Stemmer, Harder, & Hansen, 2006), and dense flows of dry granular materials (Ionescu, Mangeney, Bouchut, & Roche, 2015). Experimental works

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concerning the determination of the pressure-dependence of viscosity using mostly modified capillary rheometers can be found in the recent article by Li, Jiang, Wu, Yuan, and Li (2015).

In order to describe the pressure-dependence of the viscosity a linear law (Barus, 1891, 1893; Renardy, 2003; Georgiou, 2003; Kalogirou, Poyiadji & Georgiou, 2011) is often used:

$$\eta^* = \eta_0^*\left[1 + \beta^*(p^* - p^*_{ref})\right] \tag{1}$$

where $\eta*$ is the shear viscosity, $p*$ is the pressure, η_0^* is the viscosity at the reference pressure p^*_{ref}, and $\beta*$ is a positive parameter often referred to as the viscosity-pressure-dependence coefficient. Throughout the text a superscript star indicates a dimensional quantity; hence symbols without stars are dimensionless. In general, $\beta*$ depends on the temperature, the pressure, and the shear rate (Gustafsson, Rajagopal, Stenberg, & Videman, 2015). In isothermal Newtonian flows $\beta*$ is usually assumed to be a constant. Its values are in the range of 10–70 GPa^{-1} for lubricants (Kottke, Bair, & Winer, 2003; Tanner, 2000), 10–50 GPa^{-1} for polymer melts (Dealy & Wang, 2013; Denn, 2008; Kadjik & Van Den Brule, 1994; Tanner, 2000), and 10–20 GPa^{-1} for mineral oils (Venner & Lubrecht, 2000). Other expressions (such as the exponential law) resulting to a better fitting of available experimental data on complex fluids at different pressures have been reviewed by Málek and Rajagopal (2007) and by Housiadas (2015). Both Eqs. (1) and (2) have been employed extensively in the literature for a variety of simple flows which are important for both theoretical and experimental purposes; these include the flow due to a suddenly accelerated plate or due to an oscillating plate (Prusa, 2010; Srinivasan & Rajagopal, 2009), as well as well the laminar flows in circular tubes and straight channels (Housiadas 2015; Kalogirou et al., 2011; Poyiadji, Housiadas, Kaouri, & Georgiou, 2015; Renardy, 2003). In all cases, viscosity pressure-dependence has been found to have a substantial effect on the flow field and on features such as the skin friction factor and the pressure difference required to drive the flow.

The mass density of a liquid is expected to change at high pressures, even under isothermal conditions. Hence, the modeling of compressible flows is the subject of many works in the literature, especially in the case of complex fluids. A brief survey, for both Newtonian and viscoelastic Maxwell-type fluids, and in the framework of non-equilibrium thermodynamics, has been presented by Bollada and Phillips (2012). For Newtonian compressible liquids, a linear equation of state relating the mass density of the fluid to the total pressure is very often used (see, e.g., Venerus, 2006).

$$\rho^* = \rho_0^*\left[1 + \varepsilon^*(p^* - p^*_{ref})\right] \tag{2}$$

where ρ_0^* is the mass density of the fluid at the reference pressure p^*_{ref} and $\varepsilon*$ is the constant isothermal compressibility. Eq. (2) has been used in numerical simulations of weakly compressible liquid flows in long tubes, such as waxy crude oil (Vinay, Wachs, & Frigaard, 2006) and polymer extrusion (Taliadorou, Georgiou, & Mitsoulis, 2008).

Eqs. (1) and (2) introduce nonlinearities into the continuity and momentum equations, even for steady state, isothermal, laminar, creeping flow conditions, thus making the derivation of analytical solutions a very difficult task. Incompressible Newtonian flows with pressure-dependent viscosity have been analyzed mathematically by various investigators (see, e.g., Housiadas, Georgiou, & Tanner, 2015; Huilgol & You, 2006; Rehor & Prusa, 2016; and Vasudevaiah & Rajagopal, 2005; and references therein). Kalogirou et al. (2011) compiled analytical, two-dimensional, solutions for Poiseuille flows in a straight channel, a circular tube, and an annulus with constant inner and outer radius for an incompressible Newtonian fluid ($\varepsilon^* = 0$) obeying the linear Eq.(2) under the above-mentioned conditions. More recently, Housiadas et al. (2015) obtained perturbation solutions of the unbounded creeping flow past a sphere of a Newtonian fluid under the assumption that the shear viscosity varies either linearly or exponentially with pressure, taking the dimensionless pressure-viscosity coefficient as the perturbation parameter.

Analytical and numerical studies for compressible laminar flow of a Newtonian fluid in a tube have been conducted by van den Berg, Seldam, and van der Gulik (1993). Significant corrections to the volumetric flow-rate, compared to the predictions of the classical Poiseuille law have been revealed. The origin of those corrections was the equation of the state, i.e., from the equation that relates the mass density of the fluid to the total pressure. Analytical perturbation solutions for weakly compressible Newtonian fluids in channels and tubes, with the isothermal compressibility coefficient serving as the perturbation parameter, have been derived by Venerus and co-worker (Venerus, 2006; Venerus & Bugajsky, 2010) as well as by Taliadorou, Neophytou, and Georgiou (2009).

However, analytical studies that take into account both the compressibility and the viscosity pressure-dependence are very scarce in the literature. Recently, Poyiadji et al. (2015) derived analytical solutions for steady axisymmetric and planar Poiseuille flows of weakly compressible isothermal Newtonian liquids, using Eqs. (1)–(2) and a double regular asymptotic expansion for all the primary flow variables with small parameters the dimensionless pressure-viscosity coefficient, β, and the dimensionless coefficient of compressibility, ε (for their definitions see Section 2). Assuming that $\varepsilon \approx \beta$, they derived solutions up to second order for both parameters and analyzed the combined effects of weak compressibility and the pressure-dependent viscosity.

In the present work, the solution of Poyiadji et al. (2015) is extended by relaxing the assumption of a small β; a regular perturbation scheme is utilized in terms of εonly. By doing so, however, non-linear terms are retained into the governing equations while the solution reveals features of the flow not predicted by the double perturbation analysis. Thus, the analytical solutions derived here are valid for small values of ε and any value of β.