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Nonlinear bending and free vibration analyses of nonlocal strain gradient beams made of functionally graded material

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ABSTRACT

Size-dependent nonlinear Euler-Bernoulli and Timoshenko beam models, which account for the through-thickness power-law variation of two-constituent functionally graded (FG) materials, are derived to investigate the nonlinear bending and free vibration behaviors in the framework of the nonlocal strain gradient theory. The nonlinearity due to the stretching effect of the mid-plane of the FG beam is the source of nonlinearity of the considered bending and free vibration problems. The size-dependent equations of motion and boundary conditions are derived by employing the Hamilton's principle. The beam models contain material length scale and nonlocal parameters to consider the effects of both inter-atomic long-range force and microstructure deformation mechanism. In the case of hinged-hinged boundary conditions, the analytical solutions for the nonlinear bending deflection and free vibration frequencies of nonlocal strain gradient Euler-Bernoulli and Timoshenko beams are deduced. The influences of the through-thickness power-law variation of a two-constituent FG material and size-dependent parameters on nonlinear bending deflection and free vibration frequencies are investigated. Due to the intrinsic stiffening effect brought by the stretching effect of the mid-plane of the beam, the nonlinear bending deflections are smaller than their linear counterparts under the action of the same force, while the nonlinear vibration frequencies are higher than their linear counterparts for the same amplitude of the nonlinear oscillator. The nonlinear bending deflections and free vibration frequencies can be affected significantly by the through-thickness grading of FG materials in the beam. When the nonlocal parameter is smaller than the material characteristic parameter, the nonlinear FG beam reveals a stiffness-hardening effect. When the material characteristic parameter is smaller than the nonlocal parameter, the FG beam reveals a stiffness-softening effect.

1. Introduction

Functionally graded (FG) materials, as a new class of advanced composites, vary their microstructures from one material to another with a chosen gradient, resulting in corresponding changes in their effective material properties (such as the effective Young's modulus, the effective shear modulus and the effective material density). Traditional laminated composites may cause some unexpected problems (including high shear stress, interface cracking, and interface delamination)

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due to sudden changes of material properties at the interface of two adjacent layers. The FG materials can show many advantages over the traditional laminated composites due to the smooth variation of material properties and their excellent performances in various engineering fields of application, including higher fracture toughness, enhanced thermal resistance, enhanced corrosion resistance, improved stress spreading and inferior stress intensity factors (Jha, Kant, & Singh, 2013). To improve the performance of composite structures, the development of the FG materials are being accelerated to optimize some certain functional properties of structures by tailoring the material architecture at nano/micro scale. The rapid developments of microelectromechanical systems (MEMS) and nanoelectromechanical (NEMS) make the FG materials possible to be applied in nano/micro scaled systems. Nevertheless, the possible applications rely on a good understanding of the mechanical characteristics (such as bending and vibration) of FG structures. Significant size-dependent effects on the mechanical characteristics have been observed for small-scaled structures. Thus, the study of size-dependent effects on the bending and vibration behaviors of nano/micro-scaled FG beams is always of fundamental significance. The bending and vibration behaviors of small-scaled FG beams may be not predicted adequately by employing the classical continuum theory, some size-dependent elasticity mechanics (such as nonlocal elasticity theory, strain gradient theory and nonlocal strain gradient theory) have been successfully developed and employed to assess the size-dependent effect on the mechanical characteristics of small-scaled structures.

The nonlocal elasticity theory (Eringen, 1983), in which the stress field at a reference point is not only dependent on the strain at the reference point but also dependent on the strains at all other points in the whole body, can account for the inter-atomic long-range force. A lot of nonlocal beam models (Daneshmehr, Rajabpoor, & Hadi, 2015; Daneshmehr, Rajabpoor et al., 2014; Eltaher, Emam, & Mahmoud, 2012, 2013; Nazemnezhad & Hosseini-Hashemi, 2014; Rahmani & Pedram, 2014; Salehipour, Shahidi, & Nahvi, 2015; Şimşek & Yurtcu, 2013; Uymaz, 2013) have been developed to study the static and dynamic behaviors of FG beams based on the nonlocal elasticity theory. Recently, Ghadiri and Shafiei (2015) studied the size-dependent effect on the nonlinear vibration behaviors of a rotating nanobeam based on nonlocal elasticity theory using a differential quadrature method. Nejad and Hadi (2016a,b); Nejad, Hadi, and Rastgoo (2016) studied the size-dependent effect on the bending, buckling and free vibration problems of bi-directional FG Euler-Bernoulli nano-beams based on the nonlocal elasticity theory. These studies showed that the size independency nature of the nonlocal elasticity theory potentially plays a very significant role in studying the static and dynamical behaviors of small-scaled FG beams, and a stiffness softening effect has been observed for these nonlocal beam models.

The gradient elasticity theory (Aifantis, 1992; Mindlin, 1964) states that small-scaled materials must be modeled as atoms with higher-order deformation mechanism instead of collections of points, and the total stress should consider some additional strain gradient terms. Yang, Chong, Lam, and Tong (2002) presented a modified the gradient elasticity theory (or modified couple stress theory), in which the strain energy density needs to be considered as a function of both the strain tensor conjugated with stress tensor and the curvature tensor conjugated with couple stress tensor. By using the modified couple stress theory, Reddy (2011) developed Euler-Bernoulli and Timoshenko beams models to study the bending, vibration and buckling behaviors of FG beams. Şimşek and Reddy (2013) presented a unified beam theory containing various higher-order shear deformation beam theories as well as the Euler-Bernoulli and Timoshenko beam theories for the static bending and free vibration analysis of FG microbeams based on the modified couple stress theory. Şimşek (2014) investigated the nonlinear size-dependent static and free vibration characteristics of microbeams based on the nonlinear elastic foundation within the framework of the modified couple stress theory. Akgöz and Civalek (2014a,b) developed higher-order shear deformation beam models of FG microbeams based on the modified gradient elasticity theories. Lou and He (2015) investigated the nonlinear bending and free vibration characteristics of a simply supported FG microplate with geometry nonlinearity lying on an nonlinear elastic foundation within the framework of the modified couple stress theory and the Kirchhoff and Mindlin plate theories. Taati (2016) performed the buckling and post-buckling analysis of size-dependent FG plates based on the modified couple stress theory. Shafiei, Mousavi, and Ghadiri (2016) studied the transverse vibration of a rotary tapered axially FG microbeam based on the modified couple stress theory in the form of true spatial variation. Shafiei, Kazemi, and Ghadiri (2016) investigated the nonlinear size-dependent free vibration characteristics of a non-uniform axially FG microbeam with geometric nonlinearity based on the Euler-Bernoulli beam theory and the modified couple stress theory. Dehrouyeh-Semnani, Mostafaei, and Nikkhah-Bahrami (2016) investigated the size-dependent free vibration behaviors of FG microbeams with geometric imperfection based on the modified couple stress theory. Additionally, A lot of microstructure-dependent models (Jabbari, Nejad, & Ghannad, 2015; Khorshidi, Shariati, & Emam, 2016; Lei et al., 2015; Lou, He, Du, & Wu, 2016; Lou, He, Wu, & Du, 2016; Nejad & Fatehi, 2015; Rahaeifard, Kahrobaiyan, Ahmadian, & Firoozbakhsh, 2013; Reddy, Romanoff, & Loya, 2016; Shafiei et al., 2016; Shirazi & Ayatollahi, 2014; Şimşek, Aydın, Yurtcu, & Reddy, 2015; Sourki & Hoseini, 2016; Thai & Kim, 2013; Zhang, He, Liu, Gan, & Shen, 2014; Zhang, He, Liu, Shen, & Lei, 2015) have been also recently developed and employed to study the static and dynamical behaviors of FG beams and plates based on the modified gradient elasticity theory. These studies showed that the size independence nature of the microstructure deformation mechanism potentially plays a very significant role in studying the static and dynamical behaviors of small-scaled FG beams, and a stiffness enhancement effects have been observed in these gradient elasticity models.

From the literature discussed above, it is found that the nonlocal elasticity theory only takes into account the inter-atomic long-range force. However, similar to classical elasticity theory, the nonlocal elasticity theory states the particles are taken as mass points without considering any microstructure deformation mechanism. The gradient elasticity theory takes into account the higher-order microstructure deformation mechanism without considering any inter-atomic long-range force. The nonlocal elasticity and strain gradient theories describe two entirely different size-dependent nano/micro-mechanical