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On size-dependent Timoshenko beam element based on modified couple stress theory

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A B S T R A C T

Due to complexity of geometry of small-scale structures as well as size effect phenomenon in such scale, size-dependent finite element method can be employed as a powerful numerical technique to investigate the mechanical behavior of such structures. Recently, two distinct size-dependent Timoshenko beam elements have been proposed based on modified couple stress theory. The first beam element is a two-node element which has 3-DOF (degrees of freedom) at each node. The second beam element is also a two-node element, but it has 2-DOF at each node. Since enough verification and convergence studies have not been performed on the proposed beam elements, the present study aims to examine the accuracy, reliability and stability of aforementioned beam elements in the static bending. To that end, the cantilevered, simply supported and doubly clamped Timoshenko beams are chosen as the case studies. It is observed that the results obtained via the 6-DOF beam element are in excellent agreement with those obtained via the other solutions for the three case studies. In addition, it is found that the rate of convergence increases by ascending the influence of size-dependency. Although the 4-DOF beam element presents stable solutions, the acquired results reveal that the 4-DOF beam element is incapable of verifying the results obtained based on the other solutions for the three case studies. Moreover, it is found that the 4-DOF beam element error has an ascending trend with respect to the size-dependent shear deformation. Finally the reason for inaccuracy of the 4-DOF beam element is discussed.

1. Introduction

Yang, Chong, Lam, and Tong (2002) developed modified couple stress theory as a linear elastic constitutive law for isotropic couple stress materials based on strain gradient theory. The new higher-order elastic theory has been employed by many researchers to study the size effect phenomenon in the microstructures where the attempts of the classical models have been failed. Microbeams, as the most widely utilized continuous elements in micron and sub-micron scales, can be found in various micro-electro-mechanical systems. Hence, many researchers have developed governing equations of beam-type microstructures and have investigated mechanical behavior of such structures based on modified couple stress theory. Some of these works can be outlined as: an Euler–Bernoulli beam model for static bending analysis and comparison with experimental data by Park and Gao (2006), a microstructure-dependent Timoshenko beam model for static

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bending and free vibration analysis by Ma, Gao, and Reddy (2008), a nonlinear Euler–Bernoulli beam model for static bending, free oscillation and post buckling analysis by Xia, Wang, and Yin (2010), functionally graded Timoshenko beam models for static and free vibration analysis by Asghari, Rahaeifard, Kahrobaiyan, and Ahmadian (2011), a new model for bending analysis of composite laminated microbeams with first order shear deformation by Chen, Li, and Xu (2011), buckling analysis of axially loaded micro-scaled beams with different boundary conditions based on Euler–Bernoulli beam theory by Akgöz and Civalek (2011), nonlinear free vibration analysis of extensible functionally graded microbeams by Ke, Wang, Yang, and Kitipornchai (2012), static bending and dynamic analysis of third-order shear deformation functionally graded microbeams by Salamat-talab, Nateghi, and Torabi (2012), three-dimensional nonlinear size-dependent behaviour of Timoshenko microbeams by Ghayesh, Amabili, and Farokhi (2013), static and dynamic stability analysis of a functionally graded microbeam under electrostatic force by Abbasnejad, Rezazadeh, and Shabani (2013), static bending and free vibration analysis of functionally graded microbeams using a new higher order beam theory by Şimşek and Reddy (2013), study of thermal postbuckling behavior of size-dependent functionally graded Timoshenko microbeams by Ansari, Faghih Shojaei, Gholami, Mohammadi, and Darabi (2013), thermo-mechanical buckling behavior of functionally graded microbeams embedded in elastic medium based on sinusoidal shear deformation beam theory by Akgöz and Civalek (2014), micro-inertia effects on the dynamic characteristics of micro-beams by Fathalilou, Sadeghi, and Rezazadeh (2014), investigation of dynamic pull-in instability of geometrically nonlinear actuated microbeams by Sedighi, Chan-Gizian, and Noghreha-Badi (2014), nonlinear thermal stability and vibration analysis of pre/post-buckled temperature- and microstructure-dependent functionally graded beams resting on elastic foundation by Komijani, Esfahani, Reddy, Liu, and Eslami (2014), nonlinear static and free vibration analysis of microbeams based on the nonlinear elastic foundation using He's variational method by Şimşek (2014), nonlinear-electrostatic analysis of micro-actuated beams incorporating surface elasticity by Shaat and Mohamed (2014), three-dimensional vibration analysis of curved microbeams under fluid force induced by external flow by Tang, Ni, Wang, Luo, and Wang (2014), a functionally graded sandwich microbeam model for static bending, free vibration and buckling by Thai, Vo, Nguyen, and Lee (2015), developing a nonlinear model for cantilevered microbeams and also exploring the nonlinear dynamics i.e., frequency–response curves, phase portraits and time histories by Dai, Wang, and Wang (2015), a parametric study on nonlinear dynamics of microbeams by considering different parameters due to time-dependent longitudinal excitation load by Ghayesh, Farokhi, and Alici (2015), new models for viscoelastically damped sandwich microbeams as well as study of resonant frequency and loss factor by Dehrouyeh-Semnani, Dehrouyeh, Torabi-Kafshgari, and Nikkhah-Bahrami (2015a, 2015b), study of thermo-mechanical dynamics of perfect and imperfect Timoshenko microbeams by Farokhi and Ghayesh (2015), thermal buckling analysis of microcomposite laminated based on Euler-Bernoulli, Timoshenko and Reddy beam theories by Mohammadabadi, Daneshmehr, and Homayounfard (2015), complex sub and supercritical global dynamics of a parametrically excited microbeam subject to a time-dependent axial load with special consideration to chaotic motion by Ghayesh and Farokhi, (2015), an exact solution for vibrations of postbuckled microscale beams by Ansari, Ashrafi, and Arjangpay (2015), frequency and stability analysis of axially moving microbeams with constant velocity using different beam theories by Dehrouyeh-Semnani, Dehrouyeh, Zafari-Kolukhi, and Ghamami (2015), a new nonlinear model for dynamical performance analysis of microgyroscope (Ghayesh, Farokhi, & Alici, 2016), free vibrations and stability of spinning microbeam based on Euler–Bernoulli and Timoshenko beam theories by Ilkhani and Hosseini-Hashemi (2016), postbuckling analysis of functionally graded small scale beams under general beam theory by Akbarzadeh Khorshidi, Shariati, and Emam (2016), out-of-plane free vibration analysis of rotary tapered microbeams made of axially functionally graded materials using Euler-Bernoulli beam theory by Shafiei, Kazemi, and Ghadiri (2016b), nonlinear free vibration analysis of axially functionally graded tapered microbeams with different boundary conditions based on Euler-Bernoulli beam theory and von-Kármán's geometric nonlinearity by Shafiei, Kazemi, and Ghadiri (2016a), a new nonlinear model for coupled three dimensional microbeam as well as static bending and free oscillation analysis of a microbridge by Mojahedi and Rahaeifard (2016), nonlinear static and forced vibration analysis of CNT-based resonators under AC and DC actuations based on a fully nonlinear Euler-Bernoulli beam model by Farokhi, Païdoussis, and Misra, 2016, investigation of free flexural vibration of geometrically imperfect functionally graded microbeams with different boundary conditions by Dehrouyeh-Semnani, Mostafaei, and Nikkhah-Bahrami (2016), study of pull-in behavior of functionally graded sandwich bridges subjected to electrostatic actuation effect and intermolecular Casimir forces using Euler–Bernoulli, Timoshenko, and Reddy beam theories by Shojaeian and Zeighampour (2016), nonlinear vibration analysis of porous and imperfect functionally graded tapered microbeams by Shafiei, Mousavi, and Ghadiri (2016), investigation of large-amplitude dynamical behaviour of microcantilevers considering different sources of nonlinearity by Farokhi, Ghayesh, and Hussain (2016).

The above mentioned works deal with the size-dependent mechanical behavior of simple beam-type microstructures. In many real applications, the geometry of beam-type microstructures is not simple, therefore, employing the size-dependent beam elements to investigate the mechanical behavior of such structures sounds to be essential. Recently, two distinct size-dependent Timoshenko beam elements have been developed by Arbind and Reddy (2013) and Kahrobaiyan, Asghari, and Ahmadian (2014) for both the static and dynamic analysis. Considering only the bending deformation, both the elements are a two-node beam element, and the beam element developed by Arbind and Reddy (2013) has 3-DOF (degrees of freedom) at each node, whereas, the beam element developed by Kahrobaiyan et al. (2014) has 2-DOF at each node. In other words, the beam element proposed by Kahrobaiyan et al. (2014) with fewer DOF is capable of predicting the size-dependent mechanical behavior of Timoshenko microbeam.

Since enough verification and convergence studies have not been performed on the proposed beam elements, it seems necessary to examine the accuracy, reliability and stability of both the beam elements. In this study, using exact,