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Nonlinear free vibration of a functionally graded nanobeam using nonlocal strain gradient theory and a novel Hamiltonian approach

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In this study, a novel size-dependent beam model is proposed for nonlinear free vibration of a functionally graded (FG) nanobeam with immovable ends based on the nonlocal strain gradient theory (NLSGT) and Euler-Bernoulli beam theory in conjunction with the von-Kármán's geometric nonlinearity. It is assumed the material properties of the nanobeam changes continuously in the thickness direction according to simple power-law form. To remove the stretching and bending coupling due to the unsymmetrical material variation along the thickness, the formulation of the problem is developed based on a new reference surface. The Hamilton's principle is utilized to derive the equations of the motion and the corresponding boundary conditions. The partial nonlinear differential equation describes the nonlinear vibration of FG nanobeam is reduced to an ordinary nonlinear differential equation with cubic nonlinearity via Galerkin's approach under the assumption that the axial inertia is negligible. A closed-form solution is obtained for nonlinear frequency by the novel Hamiltonian approach, and some illustrative numerical examples are given in order to study the effects of the strain gradient length scale, the nonlocal parameters, vibration amplitude and various material compositions on the ratio of nonlinear frequency to linear frequency (the nonlinear frequency ratio).

1. Introduction

The use of miniaturized structures such as carbon nanotubes, nanorods, nanowires etc., has been increasing consistently in nano-electromechanical systems (NEMS) due to their novel mechanical (Wang, Zhang, Wang, & Tan, 2007), thermal (Dresselhaus, Dresselhaus, Charlier, & Hernandez, 2004), and electrical (Lu & Chen 2005; Hong & Myung, 2007) properties. Thus, there have been numerous experimental and theoretical studies to understand the mechanical and physical behavior of nanostructures by means of different analysis methods. One of the earliest analysis methods of nanostructures is the atomistic methods, i.e., molecular dynamic (MD) simulation. It is well-known that the atomistic methods needs long time for analysis and then they do not usually give practical solutions when the considered nanostructure consists of a large number of molecules and atoms. At this point, the use of the continuum mechanics can be an alternative way for the analysis of the large scale nanoscale structures. However, experimental and atomistic simulations results show that the

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size effect gains importance on the mechanical behavior when the dimensions of the structures are order of microns and sub-microns. In this context, the classical continuum or elasticity theory is unable to capture the size effect observed in nanostructures due to the lack of additional length scale parameter. To overcome this problem, there is a need for improving the classical continuum theory to deal with the size-dependent behavior. In order to account for size effect, the nonlocal elasticity theory proposed by Eringen (1972), which is one of size-dependent continuum theories, assumes that the stress at a point is a function of strains at all points in the continuum. Especially, the Eringen's nonlocal elasticity theory has been used to investigate the static (Peddieson, Buchanan, & McNitt, 2003; Reddy, 2007), buckling (Adali,2008; Murmu & Pradhan, 2009; Murmu & Adhikari, 2011; Pradhan & Reddy, 2011; Ansari, Sahmani, & Rouhi, 2011; Şimşek & Yurtcu, 2013), and vibration (Aydogdu, 2009; Murmu & Pradhan 2009; Şimşek, 2010; Şimşek, 2011a; Şimşek, 2011b; Eltaher, Emam, & Mahmoud, 2012; Thai, 2012; Şimşek, 2012) analysis of nanostructures in the last decade. Recently, Reddy and El-Borgi (2014), proposed the nonlinear governing equations and finite element model of Euler-Bernoulli and Timoshenko nanobeams based on the von-Karman's strain. Rahmani and Pedram (2014), have studied free vibration of FG Timoshenko nanobeam based on the nonlocal elasticity with the help of Navier solution. Ebrahimi and Salari (2015a), have presented a Navier type solution for thermal buckling and free vibration of FG Timoshenko nanobeam using the nonlocal elasticity theory. Free flexural vibration of FG Euler-Bernoulli nanobeam has been investigated by Ebrahimi and Salari (2015b), using semi-analytical differential transform method. Again, Ebrahimi and Salari (2015c), have examined thermos-mechanical vibration of FG Euler-Bernoulli nanobeam with various boundary conditions employing a semi analytical differential transform method. Nejad, Hadi, and Rastgoo (2016), have solved linear buckling problem of Euler- Bernoulli nanobeam made of two-directional functionally graded materials using generalized differential quadrature method. In a recent study by Ansari, Oskouie, Gholami, and Sadeghi (2016), free vibration behavior of piezoelectric Timoshenko nanobeams in the vicinity of postbuckling domain has been studied based on the nonlocal elasticity theory employing generalized differential quadrature method. De Rosa and Lippiello (2016), have derived the equations of motion of an embedded single-walled carbon nanotube, and they have solved the equations for free vibration problem by means of the differential quadrature method.

In spite of the fact that Eringen's nonlocal elasticity theory has been widely utilized for modeling the nanostructures, it characterizes only softening effect. However, the effect of the stiffness enhancement, which is reported from the experimental and the theoretical studies (Fleck & Hutchinson, 1993; Stolken & Evans, 1998; Lam, Yang, Chong, Wang, & Tong, 2003), cannot be taken into account by the nonlocal elasticity theory. In other words, the strain gradient and the couple stress theories and the nonlocal elasticity theory deal with the different aspects of size-dependent material behavior. As known from the previous studies, the stiffness enhancement can be incorporated by the use of the strain gradient or couple stress theories. In the literature, there are various micro-structure-dependent continuum theories proposed by the several researchers to consider this size effect (namely, the effect of the stiffness enhancement). As a first strain gradient theory (SGT), Mindlin (1965), developed a higher order strain gradient theory which includes sixteen higher-order material constants for linear isotropic materials. In 1968, Mindlin simplified this theory by considering only the first order strain gradients. In this case, there are only five independent material parameters in addition to two classical material parameters for linear isotropic materials. After this, Lam et al., (2003) developed the modified strain gradient theory (MSGT) in which the total deformation energy density is a function of the symmetric strain tensor, the dilatation gradient vector, the deviatoric stretch gradient tensor and the symmetric rotation gradient tensor. As a result of the above assumptions, the number of the non-classical material parameters is reduced from five to three. On the other hand, the main drawback of the above mentioned strain gradient theories is due to the fact that the determining the micro-structural material length scale parameters is formidable task. Accordingly, Yang, Chong, Lam, and Tong (2002), elaborated the modified couple stress theory (MCST) in which the strain energy density depends only on the strain tensor and the symmetric part of the curvature tensor. Therefore, this theory involves only one additional material length scale parameter besides two classical ones. In fact, the MCST can be considered as the special case of the modified strain gradient theory (MSGT). Based on the strain gradient and the modified couple stress theories, many studies have been performed to investigate mechanical behavior of microbeams (i.e., Park & Gao, 2006; Ma, Gao, & Reddy, 2008; Kong, Zhou, Nie & Wang, 2008; Kong, Zhou, Nie, & Wang, 2009; Ma, Gao, & Reddy, 2010; Wang, Zhao & Zhou, 2010; Şimşek, 2010; Xia, Wang & Yin, 2010; Akgöz & Civalek, 2011; Akgöz & Civalek, 2012; Salamat-talab, Nateghi & Torabi, 2012; Roque, Fidalgo, Ferreira, & Reddy, 2013; Akgöz & Civalek, 2013;). There has been still extensive research effort on the different aspects of the mechanical behavior of microbeams using the strain gradient and the modified couple stress theories. For instance, Abadi and Daneshmehr (2014a), have investigated buckling and bending of microbeams using Euler-Bernoulli, Timoshenko and higher order shear deformation theories based on the modified couple stress theory and the generalized differential quadrature method. The size-dependent buckling of laminated microbeams has been investigated by Abadi and Daneshmehr (2014b), within the framework of the modified couple stress theory. Akgöz and Civalek (2015), have proposed a size-dependent microbeam model for buckling based on the hyperbolic shear deformation theory and the modified strain gradient theory.

Another type of the strain gradient theory that contains only one material parameter was proposed by Aifantis (1992), to eliminate strain singularities from dislocation lines and crack tips and to interpret size effects. The main advantage of this simple strain gradient theory, which contains only additional material parameter, over Mindlin's strain gradient with five additional material parameters is the fact that solutions of boundary value problems can be found in terms of corresponding solutions of classical elasticity through an inhomogeneous Helmholtz equation (Aifantis, 2011). At the same time, the published literature shows that the number of studies based on the Aifantis's strain gradient theory is limited compared to the number of studies based on the Mindlin's strain gradient or the modified strain gradient theory proposed by