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Determination of pressure data from velocity data with a view toward its application in cardiovascular mechanics. Part 1. Theoretical considerations

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A B S T R A C T

The non-invasive determination of the pressure (mean normal stress) in a flowing fluid has ramifications in a variety of important problems: the flow of blood in blood vessels, flows taking place in inaccessible locations in complex internal geometries that occur in mechanical systems, etc. In this paper we discuss a rigorous new mathematical procedure for the determination of the pressure (mean normal stress) field, from data for the velocity field that can be obtained through imaging procedures such as 4D magnetic resonance imaging or echocardiography. We then use the procedure to demonstrate its efficacy by considering flows in an idealized geometry with a symmetric and asymmetric obstruction. We delineate the superiority of the method with regard to the methods that are currently in place. In Part 2 of this two part paper, we study the loss of pressure and the dissipation that occurs due to the flow of blood across a diseased valve (the pressure loss being an important indicator of the extent of the valvular disease) as well as the flow taking place in a realistic cerebral aneurysm.

1. Introduction

The ability to non-invasively determine the pressure in a flowing fluid has wide ranging technological relevance and import, an example being problems in medicine concerning the flow of biological fluids. The cardiovascular system presents several situations wherein the non-invasive determination of pressure would significantly reduce serious risks associated with invasive procedures. This two part study is concerned with the determination of the pressure field from non-invasive velocity data with a view toward determining the pressure drop across a diseased valve as a consequence of the dissipation in the fluid as it flows through the valve, the loss of pressure bearing a direct relation to the extent of the disease. While there have been some careful mathematical attempts at determining the pressure field from information for the velocity field, with regard to the Navier–Stokes fluid (see the references below), most of the studies concerning the determination of pressure from the velocity data with regard to flow across diseased valves and other related cardiovascular flow problems are based on an appeal to inappropriate governing equations, namely the Bernoulli equation, which are grossly inadequate to describe the dissipation that takes place in a flowing fluid. This is usually addressed by an ad hoc modification to the Bernoulli equation by adding a dissipation term; see

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Akins, Travis, and Yoganathan (2008). What is however required is a much more careful consideration of the viscous dissipation that takes place during the fluid flow and the resulting pressure drop. We discuss in detail the inverse problem of determining the pressure field from data for the velocity field which might or might not be known precisely. The study does not appeal to the Bernoulli equation but considers in full the Navier–Stokes equations for an incompressible linearly viscous fluid. In Part 1 of the study, in order to clarify and simplify the procedure, we consider the flow in an idealized geometry of the stenotic aortic valve. In Part 2 we will present a broader study concerned with determining the dissipation directly from the model. Part 2 will include results concerning the extent of dissipation and the corresponding pressure drop, in geometries with different levels of the severity (up to 85% in symmetric stenotic valves) and we also present some results concerning the determination of pressure in flows occurring in cerebral aneurysms in realistic geometries. In Part 1, we provide the details of the mathematical issues and the numerical procedures that are adopted to obtain the pressure field from the velocity field, while the motivation for the consideration of the problem of Valvular stenosis and Valvular disease is provided in Part 2.

While blood in small vessels exhibits shear-thinning characteristics, in a vessel of the size under consideration it can be modeled as a Navier–Stokes fluid. The resulting equations for the pressure determination can be expressed in two different ways, one as a Poisson equation for the pressure field and the other based on the Stokes equation with additional stabilization/correction term. The former method even in the weak form requires higher derivatives than the latter method. In order to ascertain the efficiency of the two methods we compute the pressure field corresponding to three types of velocity data, the first which we refer to as “fine data” wherein we have full information for the velocity field on a very fine grid, the second type of data which we refer to as “coarse data” wherein the information is available only on a rough grid with a different level of coarseness, and finally the third type of data, “data with a noise”, wherein the first two types of data are combined with an uncertainty in its values. We compare the results for the pressure field obtained from the three types of data for the velocity field using the two different methods, the pressure Poisson equation (PPE) method and the Stokes equation (STE) method. To our knowledge such a comparative study of the numerical schemes that delineates the differences in the results from the two schemes and which clearly indicates the STE method to be superior to the PPE method, for the problem on hand, have not been carried out.

We idealize the geometry of the flow to a flow in a rigid pipe with an obstruction knowing full well that the real flow domain is far more complicated with the walls being highly deformable. Our aim is to solve eventually such a complex problem for the determination of the pressure field from the data for the velocity field. While we have some experience in the numerical resolution of problems concerning fluid–solid interactions with regard to a direct determination of the velocity and pressure fields, see Hron and Madlik (2007), Razzaq, Damanik, Hron, Ouazzi, and Turek (2012), we have to develop the codes in the case of the problem of determining the pressure field from a knowledge of the velocity field. In Part 2 of the paper we consider the flow in the actual geometry that matches anatomical conditions. In this Part 1, our aim is to establish the proof of concept with regard to the mathematical formulation and the numerical procedure. We do use physiologic data for the inlet and outlet condition upstream and downstream of the obstruction (stenosis) but the flow geometry is idealized.

1.1. Determining the pressure for the flow of the Navier–Stokes fluid

The pressure loss across a diseased valve or stenotic artery is thought to be due to the dissipation that takes place during the flow across the valve. In view of this, there has been a great deal of effort expended in determining the dissipation due to the flow across a diseased valve; see Akins et al. (2008) and Dasi, Simon, Sucosky, and Yoganathan (2009). However, all these efforts are totally ad hoc and appeal to procedures that are clearly not suited for evaluating the dissipation. This is because the approaches used to non-invasively ascertain energetic losses are arbitrary modifications of models that actually describe conservation of energy; the very essence of a stenosis is that it poses an impedance to flow, dissipating the energy of flowing blood. The walls of the blood vessel are in fact inhomogeneous and anisotropic, and more importantly viscoelastic. Thus, in order to determine the flow characteristic of blood across a diseased valve, it is necessary to solve the equations that couple the motion of the wall boundary as well as the equations governing the flowing blood. The problem is exceedingly difficult in view of the fact that the flow domain is changing with time and the domain itself can only be determined by solving the coupled system of equations. It is our aim to eventually study such a problem. Here we study flows in a geometry with rigid boundaries.

In very small blood vessels such as capillaries as the diameter of the red blood cells are of comparable or even larger diameter than that of the blood vessel, even using a continuum fluid model for blood is totally inappropriate. However, in the problem of flow across a diseased valve or arterial stenosis the flow of blood can be well described by the classical incompressible Navier–Stokes fluid. Even if we consider the fluid flowing in a vessel that has a rigid boundary, the problem is quite challenging as we have to solve the problem wherein locally the Reynolds number is very large, the geometry of the flow domain is complex, and the flow is three dimensional and unsteady.

Let us suppose that blood can be described by the Navier–Stokes constitutive relation, that is

$$\mathbf{T} = -p\mathbf{I} + 2\mu\mathbf{D}, \tag{1}$$

where $\mathbf{T}$ is the Cauchy stress, $p = -\frac{1}{3}(\operatorname{tr}\mathbf{T})$ is the mean normal stress that is usually referred to as the mechanical pressure (see Rajagopal, 2015 for a detailed discussion of the notion of “pressure”), μ is the dynamic viscosity, $\mathbf{v}$ is the velocity and $\mathbf{D} = \frac{1}{2}\left(\nabla\mathbf{v} + \left(\nabla\mathbf{v}\right)^T\right)$. Since the fluid is incompressible, it can only undergo isochoric motions so that the constraint

$$\operatorname{tr}\mathbf{D} = \operatorname{div}\mathbf{v} = 0 \tag{2}$$

is satisfied. We shall assume that the fluid is homogeneous and hence the balance of mass reduces to the density ϱ being a constant everywhere.