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Thermo-elasto-dynamic analysis of axially functionally graded non-uniform nanobeams with surface energy

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ABSTRACT

This paper deals with transverse vibration of axially functionally graded tapered nanoscaled beams acted upon by a longitudinal temperature gradient. Using surface elasticity theory of Gurtin–Murdoch, the equations of motion of the nanostructure are displayed based on the hypotheses of the Rayleigh, Timoshenko, and higher-order beam theory. Due to the variation of the material and the cross-section along the nanobeam, seeking an analytical solution to the resulting governing equations is a very cumbersome job. To conquer this difficulty, reproducing kernel particle method is proposed, and the natural frequencies of the thermally affected nanostructure are numerically calculated. Subsequently, the roles of the slenderness ratio, temperature gradient, diameter of the nanobeam, and variation of both the cross section and the material property along the length of the nanobeam on its free dynamic response are investigated. In each parametric study, the effects of both surface energy and shear deformation on the natural frequencies are addressed and explained.

1. Introduction

Recently, micro-/nano-scaled structures made from functionally graded materials (FGMs) have been proposed as building blocks of micro-/nano-electromechanical systems (MEMS/NEMS) (Fu, Du, Huang, Zhang, & Hu, 2004; Witvrouw & Mehta, 2005) as well as shape memory alloy and thin films (Yoshimura, Suchanek, Watanabe, Sakurai, & Abe 1998; Bogdanski et al., 2002; Fu, Du, & Zhang, 2003; Sioh, 2010). The potential usage of functionally graded nanosensors and nanoactuators is also under investigation. The mechanical behaviors of most of MEMS/NEMS are commonly the same as those of beam-like structures. Therefore, realizing of deformation of functionally graded microbeams/nanobeams leads to better understanding of mechanical response of the above-mentioned systems.

In FGMs, the composition and structure of the constituents would gradually change over volume, resulting in variation of the material's properties. The basic concept is to provide composite materials by specific alteration the microstructure from one material to another one. Such a fact enables the FGM to have the best of its constitutive materials in the needed directions. These materials are commonly designed for particular functions such as high temperature, fracture, fatigue, stress corrosion cracking, and corrosive resistance. In functionally graded nanobeams, the material properties can be allowed to vary in both longitudinal and transverse directions for the considered jobs. Herein, not only the material properties of the nanobeam of our concern, but also its geometry data would vary across the length. Such a nanostructure is called *axially*

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functionally graded non-uniform nanobeam (AFGNNB). The main purpose of this study is to examine transverse vibrations of these nanostructures when they are acted upon by a longitudinal thermal field. To this end, appropriate surface energy-based beam models are developed and their capabilities in capturing the natural frequencies are discussed in some detail.

At the nanoscale, the ratio of the surface area to the bulk volume is large enough that the role of elastic strain energy of the surface layer in the mechanical behavior of the nanostructure becomes important. The surface elasticity theory of Gurtin and Murdoch (Gurtin & Murdoch, 1975; 1976; 1978) is one of the successful theories to include such effect in the continuum-based modeling of nanoscaled structures. In brief, this theory explains that the surface is a very thin layer of negligible thickness which has been tightly attached to the underlying bulk. It implies that the displacements of the surface layer are identical to those of the bulk at the vicinity of the surface. However, the mechanical behavior of the surface layer is completely different from that of the bulk. The surface elasticity theory of Gurtin–Murdoch introduces residual surface stress plus to two Lame's constants to the constitutive relations of the surface layer. The magnitude of these parameters are commonly determined by comparing the obtained results by the surface elasticity-based model and those of appropriate atomistic-based models (Chen, Shi, Zhang, Zhu, & Yan, 2006; Shenoy, 2005) or experiments (Jing et al., 2006; Zheng, Cao, Li, Feng, & Wang, 2010). To investigate the role of surface stress in the elastic properties of nanostructures, Wang, Zhao, and Huang (2010) presented the surface elasticity-based formulations of elastic solids for both Lagrangian and Eulerian contexts. To date, the surface elasticity theory has been widely employed in statics (Fu, Zhang, & Jiang, 2010; Jiang & Yan, 2010; Mahmoud, Eltaher, Alshorbagy, & Meletis, 2012; Yan & Jiang, 2011), free dynamics (Eltaher, Emam, & Mahmoud, 2012b; Gheshlaghi & Hasheminejad, 2011; Kiani, 2014d; 2015c; Wang & Feng, 2010; Wang, 2010), forced vibrations (Ansari, Mohammadi, Shojaei, Gholami, & Sahmani, 2014b; Kiani, 2014a), buckling (Kiani, 2015a; 2015b; Li, Song, Fang, & Zhang, 2011; Wang & Feng, 2009; Wang, 2012), and postbuckling (Ansari, Mohammadi, Faghih Shojaei, Gholami, & Sahmani, 2014a; Sahmani, Bahrami, & Aghdam, 2016) of nanoscale beam-like structures. In all these carried out studies, the cross-section of the nanobeam is uniform and the material's properties are constant across the nanobeam's length.

Concerning vibrations of nanobeams with non-uniform cross section, Malekzadeh and Shojaee (2013) studied nonlinear flexural vibration of elastically supported nanobeams accounting for both surface and nonlocality effects. To this end, both the Euler–Bernoulli and Timoshenko beam theories were implemented. By using Hamilton's principle in conjunction with surface elasticity theory of Gurtin–Murdoch and nonlocal continuum theory of Eringen, the governing equations were derived and solved via differential quadrature method. Based on the hypotheses of the Euler–Bernoulli beam by considering the von-Karman strain them and surface energy, On, Altus, and Tadmor (2010) investigated static deformation of nano-scaled non-uniform beams. For special variations of materials along the length, the obtained results by the suggested continuum-based model were checked with those of an atomistic model. Regarding mechanical analysis of functionally graded nanobeams, Sharabiani and Yazdi (2013) investigated nonlinear free vibration of nanobeams with allowance of material variation across the thickness using the surface energy-based Euler–Bernoulli beam theory. In another work, Hosseini-Hashemi and Nazemnezhad (2013) examined nonlinear vibration of thin nanobeams analytically. The nonlinear frequencies of the nanostructure were calculated using multiple scale method, and the influences of the length, volume fraction index, amplitude ratio, and thickness on the natural frequencies were addressed. There exist also several works on transverse vibrations of functionally graded uniform nanobeams using nonlocal elasticity (Eltaher, Alshorbagy, & Mahmoud, 2013; Eltaher, Emam, & Mahmoud, 2012a; Hosseini-Hashemi, Nahas, Fakher, & Nazemnezhad, 2014; Kiani, 2014b; Pedram, 2014). A brief survey of the literature reveals that transverse vibrations of AFGNNBs in the presence of temperature gradient have not been explored thoroughly. Further, the roles of surface effect and shear deformation on their natural frequencies have not been answered convincingly yet.

Due to the variation of both geometry and material properties across the nanobeam's length, the resulted equations of motion for the developed models are highly spatial dependent. As a result, seeking for an exact or even an analytical solution to them is a very problematic job. To overcome these difficulties, reproducing kernel particle method (RKPM) is adopted. This efficient meshless methodology employs higher-order shape functions and thereby, it enables us to use that for the problems suffer from highly gradient field in partial part or whole part of the domain. Until now, RKPM has been widely used for dynamic analysis of macrobeams (Aluru, 1999; Kiani & Nikkhoo, 2012; Zhou, Zhang, & Zhang, 2005) and nanobeams (Kiani, 2014c). In the present work, the strong form of the equations of motion of each model is obtained in the context of the surface elasticity-based theory of Gurtin–Murdoch. Thereafter, the Galerkin-based RKPM approach is applied to these equations to explore the transverse dynamic behavior of thermally affected AFGNNBs.

Herein, by employing Rayleigh beam theory (RBT), Timoshenko beam theory (TBT), and higher-order beam theory (HOBT) and evaluating the surface energy for each model, the dimensionless equations of motion are constructed using the Hamilton's principle. Due to variation of the material properties of the bulk and the surface layer along the length of the nanobeam, finding an analytical solution to the problem even for simply supported boundary condition is a cumbersome task. To overcome this dilemma, an efficient meshless methodology is proposed and the essential boundary conditions are enforced by the corrected collocation approach. Subsequently, the natural frequencies associated with the transverse vibration of each model are calculated. The roles of the nanobeam's diameter, slenderness ratio, temperature gradient, power-law index, and surface energy on the natural frequencies of AFGNNBs are addressed. Specifically, the influences of the shear deformation and surface energy on the obtained results are explained and discussed. This work can be regarded as an appropriate benchmark for transverse vibration analysis of more complex nanosystems