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Size-dependent behavior of functionally graded anisotropic composite plates

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ABSTRACT

Based on the modified couple-stress theory, the size-dependent behavior of the functionally graded anisotropic elastic composites is analytically investigated when the load is applied on the top surface of the composite plate. The functionally graded material is assumed to be exponential in the thickness direction of the plate. By expanding the solutions of the displacements in terms of the two-dimensional Fourier series, the final governing equations are reduced to an eigenvalue and eigenvector problem. The exact solutions of the elastic fields with the modified couple-stress effect are derived under two kinds of boundary conditions. The classical elastic solutions are reduced from the present solutions as a special case. Numerical examples show the effect of the functional gradient factor and the material length-scale on the elastic fields along the thickness direction of the plate. Some important features observed in this paper could be useful in designing the functionally graded laminated composites with size-dependency.

1. Introduction

Small-scale structures with micron and nanometer scales have been widely used in micro/nanotechnology and devices, such as biosensors, atomic force microscopes (AFMs), nano-wires, microactuators, and micro- and nano-electromechanical systems (MEMS and NEMS) (Li, Bhushan, Takashima, Baek, & Kim, 2003; Pei, Tian, & Thundat, 2004). Experimental revealed that the size-dependent effect plays an important role in these small-scale structures (Chong & Lam, 1999; Fleck, Muller, Ashby, & Hutchinson, 1994; Lam, Yang, Chong, Wang, & Tong, 2003; Ma & Clarke, 1995; McFarland & Colton, 2005; Stölken & Evans, 1998). To describe the size-dependency of small-scale structures, Cosserat and Cosserat (1909), Toupin (1962), and Mindlin and Tiersten (1962) developed the couple-stress theory that contains two additional material constants for isotropic materials. The two additional constants are difficult to determine and are dependent on the underlying microstructure of the materials. By introducing the concept of the representative element and defining the force and couple applied to a single material particle, Yang, Chong, Lam, and Tong (2002) proposed the modified couple-stress theory which involves only one additional material parameter. Since then, the modified couple-stress theory has been applied by many researchers to study the size-dependent effect for the small-scale structures.

In engineering practice, micro- and nano-structures are in the form of beams and plates. Based on the modified couple-stress theory, several size-dependent beam models were established, such as the Bernoulli-Euler beam model

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(Kong, Zhou, Nie, & Wang, 2008; Park & Gao, 2006; Xia, Wang, & Yin, 2010), Timoshenko beam model (Asghari, Kahrobaiyan, & Ahmadian, 2010; Ma, Gao, & Reddy, 2008), composite laminated beam model (Chen, Chen, & Sze, 2012; Chen, Li, & Xu, 2011; Mohammad Abadi & Daneshmehr, 2014; Roque, Fidalgo, Ferreira, & Reddy, 2013) and the nonlinear microbeam model (Farokhi & Ghayesh, 2016; Mojahedi & Rahaeifard, 2016). Tsiatas (2009) developed a Kirchhoff plate model for the static analysis of microplates by utilizing the modified couple stress theory. Jomehzadeh, Noori, and Saidi (2011) used this model to analyze the vibration of microplates. Wang, Zhou, Zhao, and Chen (2011) developed a size-dependent Kirchhoff microplate model based on the strain gradient elasticity theory. Ma, Gao, and Reddy (2011) established a Mindlin plate model based on the modified couple stress theory. Chen, Xu, and Li (2012) developed a composite laminated plate model based on the modified couple stress theory. Ashoori Movassagh and Mahmoodi (2013) presented a Kirchhoff microplate model based on the modified strain-gradient elasticity. Roque, Ferreira, and Reddy (2013) studied the bending of Mindlin microplates with a modified couple stress theory and a meshless method. Chen and Li (2014) proposed a modified couple stress theory for anisotropic elasticity and developed a composite laminated Kirchhoff plate model. More recently, Hosseini and Bahaadini (2016) analyzed the size-dependent stability of cantilever micro-pipes based on the modified strain-gradient theory.

Functionally gradient material (FGM) is a class of composite materials that has a gradual variation of material properties from one surface to the other. The distinguishing feature of the FGM is the non-uniform microstructures with continuously graded macroproperties. An FGM can be defined by the variation in the volume fractions. The power-law function (Wang & Pan, 2011; Yung & Munz, 1996) and exponential function (Delale & Erdogan, 1983; Erdogan & Chen, 1998; Jin & Batra, 1996; Jin & Noda, 1994; Pan & Han, 2005) are commonly used to describe the variation of properties of FGM. For the exponential variation, the material properties, say the elastic coefficients, vary according to formula: $C_{ik}(z) = C_{ij}e^{\eta z}$. By adjusting the elastic moduli C_{ij} and the exponential factor η, a reasonable approximation to the distribution of the material property as functions of z can be derived (Delale & Erdogan, 1983). Actually, experimental results (Li, Lambros, Cheeseman, & Santare, 2000; Rousseau, Chalivendra, Tippur, & Shukla, 2010) showed good agreements with the theoretical solution (Erdogan & Wu, 1997; Gu & Asaro, 1997; Guo & Noda, 2007) based on the assumption of exponential functions for FGM. Comparing to other functions such as linear and power-law functions, the exponential-function variation is much more convenient and effective in solving a given problem (e.g., Pan & Han, 2005). More recently, based on a simple micromechanical method in micromechanics (i.e., Kachanov & Sevostianov, 2013), Zhang, Waksmanski, Wheeler, Pan, and Larsen (2015) developed a very useful approach for estimating the effective properties of the FGM with exponential variation.

Recently, FGMs were widely applied to micro- and nano-structures such as thin films in the form of shape memory alloys and in MEMS and NEMS and AFMs (Craciunescu & Wuttig, 2003; Fu, Du, & Zhang, 2003; Fu, Du, Huang, Zhang, & Hu, 2004; Lee et al., 2006; Witvrouw & Mehta, 2005). The size-dependent static and vibration behavior of micro-beams made of FGMs was analytically investigated (Asghari, Ahmadian, Kahrobaiyan, & Rahaeifard, 2010; Asghari, Rahaeifard, Kahrobaiyan, & Ahmadian, 2011). Based on the modified couple-stress theory, Reddy (2011) developed a microstructure-dependent couple-stress theory of FGM beams. Ke and Wang (2011) and Ke, Wang, Yang, and Kitipornchai (2012) studied the dynamic stability and nonlinear vibration of FGM micro-beams, respectively. Ansari, Gholami, Faghih Shojaei, Mohammadi, and Sahmani (2013) analyzed the bending, buckling and free vibration of an FGM Timoshenko micro-beam based on the strain-gradient theory. Shafiei, Kazemi, and Ghadiri (2016) studied the size-dependent vibration of an axially FGM microbeam based on the modified couple-stress theory. Based on the modified couple-stress and sinusoidal shear-deformation theories, Thai and Thuc (2013) presented a size-dependent model for bending and free vibration analysis of FGM plates. A sigmoid FGM nanoplate over on an elastic medium was analyzed by Jung, Han, and Park (2014) based on the modified couple-stress theory. While Ke, Yang, Kitipornchai, and Wang (2014) investigated the axisymmetric postbuckling of FGM annular microplates, Shaat, Mahmoud, Gao, and Faheem (2014) proposed a Kirchhoff plate model using the modified couple-stress theory to study the bending behavior of nano-sized plates, including surface energy and microstructure effects. More recently, Li and Pan (2015) investigated the static bending and free vibration of an FGM piezoelectric microplate based on the modified couple-stress theorem, and Gao and Zhang (2016) proposed a non-classical Kirchhoff plate model by using the modified couple-stress theory, the surface elasticity theory and the two-parameter elastic foundation model. Taati (2016) analyzed the buckling of a FGM microplate based on the modified couple-stress theory.

As discussed above, previous works on modified couple-stress effect are limited to the beam and thin plate structures. To our best knowledge, the anisotropic three-dimensional (3D) composite plate problem with modified couple-stress effect has not been reported in the literature. This motivates the present study where we consider the size-dependent behavior of the 3D FGM anisotropic elastic composite plate analytically under the surface load based on the modified couple-stress theory. The material property of FGM is assumed to be exponential function in the thickness direction of the plate.

This paper is organized as follows: In Section 2, we describe the problem along with the associated basic equations based on the modified couple-stress theory. In Section 3, we derive the general solution of the FGM anisotropic composite plate based on the modified couple-stress theory. Numerical examples are presented and discussed in Section 4 and conclusions are drawn in Section 5.

2. Problem description and basic equations

Let us consider an anisotropic and rectangular 3D FGM composite plate with the horizontal dimensions L_x and L_y and thickness H. The four sides of the FGM plate are simply supported and the material properties vary exponentially in the