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# Determination of entropy for special classes of temperature-rate and strain-rate dependent inelastic materials

A. Khoeini, A. Imam*

*Department of mechanical and aerospace engineering, Science and Research branch, Islamic Azad University, Tehran, Iran*

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ABSTRACT

With the use of a reduced energy equation and the Green–Naghdi dissipation inequality, the entropy function is determined in terms of the Helmholtz free energy and other functions that are known or can be determined for certain classes of materials with temperature-rate and strain-rate dependent thermomechanical constitutive response functions.

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## 1. Introduction

The first law of thermodynamics, also known as the balance of energy, asserts the equivalence of the mechanical energy and heat and their conservation for every material volume of the body in every thermomechanical process. In the first law, it is assumed that mechanical energy can be transformed into heat and conversely, with no restrictions placed on such transformations. Experimentally it is known that transformation of mechanical energy into heat (for example, through friction) is not limited by any restrictions, however, the reverse transformation of heat into mechanical energy has certain limitations. This fact is presented in the form of a number of statements commonly referred to as "the second law of thermodynamics". It is often stipulated that the various statements of the second law are equivalent (e.g., Pippard, 1966; Planck, 1945; Schmidt, 1949; ter Haar & Wergeland, 1966; Zemansky, 1996).

However, efforts to place exact restrictions on the exchange of heat and mechanical energy in the context of continuum mechanics have been rather controversial, although certain measure of agreement related to specific materials has been achieved. Many of the differences appear to be due to the concept of entropy; none of the statements of the second law involve entropy or appear to lead to its existence, except in special cases. Many researchers in continuum mechanics have postulated the existence of a scalar field, called "entropy", and so far a number of different approaches have been proposed which incorporate the concept of entropy and lead to a form of the second law of thermodynamics (Naghdi, 1980). Others have attempted to show the existence of the entropy for certain classes of materials (Casey, 2005, 2011).

The role of the second law becomes particularly important with respect to the behavior of dissipative materials wherein restrictions are placed on the constitutive response functions of the material. One of the earliest such attempts is due to Coleman and Noll (1963) where they proposed the Clausius–Duhem inequality as a statement of the second law for

* Corresponding author.
*E-mail addresses:* khoeini.ali@gmail.com (A. Khoeini), aimam@srbiau.ac.ir (A. Imam).

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every admissible thermomechanical processes. Subsequently, with the help of this inequality restrictions were placed on the constitutive relations for some classes of materials (e.g., Acharya & Shawki, 1996; Carlson, 1972; Green & Naghdi, 1965, 1966). Some restrictions of this kind do seem to embody concepts in various statements of the second law, making the Clausius–Duhem inequality widely regarded as representing the second law of thermodynamics. Although the results obtained by using this approach often receive widespread acceptance, the inequality has been the subject of some criticism (e.g., Day, 1972, 1977; Green & Naghdi, 1977, 1978a). Some writers in the field of thermomechanics assume the validity of the Clausius–Duhem inequality with the entropy assumed to be a primitive quantity despite the fact that no standard of entropy exists, nor does any procedure exist by which the entropy of a given system could be compared with that of an entropy standard, if such existed (Rivlin, 1986). Some other writers assume the entropy to be a functional of the process history up to the present instant (Rivlin, 1986).

If the constitutive response functions are independent of temperature-rate, by employing the Clausius–Duhem inequality, entropy as a function can be determined in terms of the Helmholtz free energy. But if the constitutive response functions are dependent on the time derivative of temperature, entropy cannot be determined from the Clausius–Duhem inequality and remains undetermined.

Green and Laws (1972) proposed a generalization of the Clausius–Duhem inequality wherein the absolute temperature was replaced by a function requiring a constitutive equation, which was later called thermodynamic temperature by some writers (Suhubi, 1975). They assumed that the thermodynamic temperature is positive and at states of equilibrium reduces to the absolute temperature. In some works through the use of this inequality, the entropy is determined in terms of the thermodynamic temperature and the Helmholtz free energy for materials whose constitutive response functions include dependence on the first and higher time derivatives of the temperature (Batra, 1976; Green & Lindsay, 1972; Suhubi, 1975, 1982). However, no prescription is given for determining the thermodynamic temperature itself much reminiscent of the case where the entropy was introduced as a primitive quantity.

Green and Naghdi (1977, 1978a) proposed a dissipation inequality embodying certain aspects of the second law of thermodynamics which was not subject to the apparent limitations of the Clausius–Duhem inequality. Some researchers employed this inequality to place restrictions on the constitutive response functions (e.g., Green & Naghdi, 1978b, 1978c, 1984; Rubin, 1986, 2015; Johnson & Imam, 1999).

The aim of the present paper is to determine the entropy for some classes of temperature-rate and strain-rate dependent inelastic materials. For this purpose the reduced energy equation and the Green–Naghdi inequality are employed to obtain an expression for the entropy in terms of the Helmholtz free energy and other response functions that are known or otherwise can be determined.

## 2. Basic equations

### 2.1. Conservation laws and energy equation

Consider a finite body $B$ with material points $X$, and identify the material point $X$ in $B$ with its position $\boldsymbol{X}$ in a fixed reference configuration. A motion of the point $\boldsymbol{X}$ at any time $t$ is defined by a differentiable vector function $\boldsymbol{\chi}$ in the form of $\boldsymbol{x} = \boldsymbol{\chi}(\boldsymbol{X}, t)$. In the present configuration at time $t$, the body $B$ occupies a region of space $R$ bounded by a closed surface $\partial R$. Similarly, in the present configuration, an arbitrary subset $p \subseteq B$ of the body $B$ occupies the region $p$, bounded by a closed surface $\partial p$. Then, the deformation gradient tensor $\boldsymbol{F}$, the velocity vector at time $t$, the right Cauchy–Green deformation tensor $\boldsymbol{C}$, the Lagrangian strain tensor $\boldsymbol{E}$ are given by

$$\boldsymbol{F} = \frac{\partial \boldsymbol{\chi}}{\partial \boldsymbol{X}}, \qquad \boldsymbol{v} = \frac{\partial \boldsymbol{\chi}}{\partial t} = \dot{\boldsymbol{x}}, \qquad \boldsymbol{C} = \boldsymbol{F}^{\boldsymbol{T}}\boldsymbol{F}, \qquad \boldsymbol{E} = \frac{1}{2}(\boldsymbol{C} - \boldsymbol{I}) \tag{2.1}$$

where $\boldsymbol{F}^{\boldsymbol{T}}$ denotes the transpose of $\boldsymbol{F}$.

The conservation laws of classical continuum mechanics for mass, momentum and moment of momentum which can be stated for every material part of $B$ occupying a region $p$ in the present configuration, under suitable continuity assumptions, have the local form

$$\dot{\rho} + \rho \; div \; \boldsymbol{v} = 0 \tag{2.2a}$$

$$div \; \boldsymbol{T} + \rho \; \boldsymbol{b} = \rho \dot{\boldsymbol{v}} \tag{2.2b}$$

$$\boldsymbol{T} = \boldsymbol{T}^{T} \tag{2.2c}$$

where the following relationship holds

$$\boldsymbol{t} = \boldsymbol{T}\boldsymbol{n} \tag{2.2d}$$

In Eqs. (2.2a) to (2.2d), $\rho$ is the mass density in the present configuration, $\boldsymbol{b}$ is the external body force per unit mass acting on the body $B$ in the present configuration, $\boldsymbol{t}$ is the stress vector, $\boldsymbol{n}$ is the outward unit normal to the surface $\partial p$, $\boldsymbol{T}$ is

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