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Analytical solutions for the size dependent buckling and postbuckling behavior of functionally graded micro-plates

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ABSTRACT

In this study, the buckling and postbuckling analysis of FG micro-plates under different kinds of traction on the edges is investigated based on the modified couple stress theory. The static equilibrium equations of an FG rectangular micro-plate as well as the boundary conditions are derived using the principle of minimum total potential energy. The analytical solutions are developed for three case studies including: simply supported micro-plates subjected to uniform transverse load and biaxial tractions, clamped-simply supported micro-plates under uniform transverse load and axial traction, and simply supported micro-plates subjected to shear traction. All plate properties except the length scale parameter are assumed to vary through the thickness according to the simple power law. The numerical results present the effects of the aspect ratio, material length scale parameter and power index on the critical values of traction and the curves of static equilibrium paths. Findings indicate that the length scale parameter is the most effective factor on the critical buckling values. Meanwhile, the aspect ratio has the most influence on the forms of the static equilibrium paths.

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1. Introduction

1.1. The size dependent behavior of micro-structures

Micro-plates, the plates whose characteristic sizes are in the orders of micron and sub-micron, play a prominent role in micro- and nano-electromechanical systems (MEMS and NEMS) such as atomic force microscopes, micro-actuators, and micro-pumps (Hua et al., 2007; Pelesko, & Bernstein, 2003). The size dependent behavior, which observed experimentally in small-scale structures (Lam, Yang, Chong, Wang, & Tong, 2003; McFarland & Colton, 2005), has major effects on the mechanical parameters of design, e.g. natural frequencies, maximum deflections and buckling loads. Since the plate models based on the classical continuum theories are not able to interpret the experimentally-detected small-scale effects in micro-scale elements, several non-classical continuum theories such as the couple stress (Toupin, 1962), nonlocal (Eringen, 1968 & 2002), modified strain gradient (Lam et al., 2003) and modified couple stress (Yang, Chong, Lam, & Tong, 2002) theories have been developed to capture the size effect. Toupin (1962) proposed the couple stress theory in which the presence of the couple stress tensor is taken into account beside the classical force stress tensor. This theory contains two higher order material length scale parameters in addition to the two Lame constants. Considering symmetry of the couple stress tensor, Yang et al. (2002) suggested a simple couple stress theory whose length scale parameters were reduced to one parameter. The theory developed by this consideration is called the modified couple stress theory. In recent years, the non-classical continuum theories have been broadly utilized to formulate and

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investigate the size-dependent mechanical behavior of micro-beams and plates. Some examples of such works are mentioned here: Ma, Gao, and Reddy (2008); Taati, Molaei, and Basirat (2014); Taati, Molaei, and Reddy (2014); Taati, Nikfar, and Ahmadian (2012); Tsiatas (2009).

1.2. Functionally graded materials

Functionally graded materials (FGMs) are composed of two or more materials in which the properties are varied as a continuous function of position along certain direction(s) from one point to another. The combination of a metal phase and a ceramic phase can give both the desired mechanical and thermal properties. Hence, the most of these materials consist of ceramic and metalphases. Recently, the numerous demands have been developed for reaching the multilayer MEMS with variable properties in the thickness. For example, the processes were defined to create an FG layer in micron size with the desired electrical and mechanical properties at its bottom and top surfaces (Witvrouw & Mehta, 2005). Hence, many researchers have been focused on the analysis of FG microstructures.

1.3. Studies accomplished on the FG micro-structures

In this section, some studies on the general analyses of FG microstructures are reviewed. Molaei, Ahmadian, and Taati (2014) investigated the effect of thermal wave propagation on thermo-elastic behavior of FG micro-slabs symmetrically surface heated using analytical solution. Their findings indicated the non-Fourier heat conduction model for microstructures has significant influence on the dynamic temperature and stress field. Molaei, Taati, and Basirat (2014) determined the distribution of transient temperature and associated thermal stresses in an FG micro-slab symmetrically heated on both sides by separation of the variables scheme. Further, effects of inhomogeneity parameter and end support on the distribution of the temperature, displacement and stress were investigated. Ke and Wang (2011) studied the dynamic stability of FG micro-beams based on the modified couple stress theory and Timoshenko beam model. The differential quadrature (DQ) method was employed to convert the governing differential equations into a linear system of Mathieu–Hill equations. Kahrobaiyan, Rahaeifard, Tajalli, and Ahmadian (2012) proposed a strain gradient model for the static and free-vibration behavior of FG Euler–Bernoulli beams. The model included five equivalent length scale parameters which can be extracted from the length scales of material constituents. Reddy and Berry (2012) developed a microstructure-dependent nonlinear theory for axisymmetric bending of circular plates, which accounts for through-thickness power-law variation of a two-constituent material, using Hamilton's principle. This formulation was based on the modified couple stress theory, power-law variation of the material, temperature-dependent properties, and the von Kármán geometric nonlinearity. Reddy and Kim (2012) formulated a general third-order model of FG plates with microstructure-dependent length scale parameter and the von Kármán nonlinearity. Their model considered the temperature dependent properties of the constituents in the functionally graded material. Furthermore, the modified couple stress theory was used to capture the size effect, in which the length scale parameter was assumed to be constant through the thickness. Thai and Choi (2013) presented size dependent models for bending, buckling, and vibration of functionally graded Kirchhoff and Mindlin plates utilizing the modified couple stress theory. The numerical results demonstrated that the inclusion of the small scale effect leads to a reduction in the deflection magnitude and an increase in the amplitudes of buckling load and frequencies. Asghari and Taati (2013) extended a size-dependent model for mechanical analyses of FG micro-plates based on the modified couple stress theory. The governing differential equations of motion and the boundary conditions were derived for FG micro-plates with arbitrary shapes employing the Hamilton principle.

1.4. Studies accomplished on the buckling and postbuckling behavior of micro-structures

Here, the some works accomplished on the buckling and postbuckling analysis of micro-structures are mentioned. For instance, Mohammad-Abadi and Daneshmehr (2014) studied buckling behavior of three micro-beam models including Euler–Bernoulli beam theory (EBT), Timoshenko beam theory (TBT) and Reddy beam theory (RBT) based on modified couple stress theory. To examine the effect of boundary conditions, three kinds of boundary conditions i.e. hinged–hinged, clamped–hinged and clamped–clamped boundary conditions were assumed. Ke, Yang, Kitipornchai, and Wang (2014) presented the axisymmetric postbuckling of FG annular micro-plates using the modified couple stress theory, Mindlin plate theory and von Kármán geometric nonlinearity. Material properties were considered to be graded in the thickness direction according to Mori–Tanaka homogenization method. By utilizing the physical neutral plane, the bending-extension coupling was eliminated in both nonlinear governing equations and boundary conditions of the FG micro-plate. Daneshmehr, Rajabpoor, and Pourdavood (2014) provided a nonlocal higher order plate model for stability analysis of nano-plates subjected to biaxial in plane loadings. It was assumed that the properties of the FG nano-plate follow a power law form through the thickness. The governing equations and corresponding boundary conditions were obtained with the aid of minimum total potential energy principle. The Generalized differential quadrature (GDQ) method was employed to solve the size dependent governing equations according to the higher order shear deformation plate theories. Akgöz and Civalek (2014) performed thermo-mechanical size-dependent buckling analysis of embedded functionally graded (FG) micro-beams based on the sinusoidal shear deformation beam and modified couple stress theories. The Winkler elastic foundation model was applied to simulate the interaction between the FG micro-beam and the elastic medium. The static equilibrium equations and corresponding boundary conditions were derived by use of the principle of minimum potential energy. Mohammadabadi, Daneshmehr, and Homayounfard (2015) investigated the thermal effect