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Thermodynamic formulation of a viscoplastic constitutive model capturing unusual loading rate sensitivity

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ABSTRACT

Metallic materials and alloys may exhibit serrated flow, rate insensitivity or negative rate sensitivity in a certain range of strain, strain rate and temperature. This behavior is referred to as dynamic strain aging phenomenon. A rate-dependent nonlinear kinematic hardening model was proposed by Ho (2008) in order to model positive, zero, negative rate sensitivity of flow stress and other rate-dependent inelastic behavior in a consistent way. But it has not been demonstrated that the proposed model can be accommodated by the framework of thermodynamics of irreversible processes. The present contribution thus aims at thoroughly examining the thermodynamic consistency of the constitutive model. The evolution equations for the internal state variables are derived through the introduction of two potentials, the thermodynamic potential and the dissipation potential.

1. Introduction

In general, metals and alloys show time-dependent deformation behavior such as loading rate sensitivity, creep and relaxation. The inelastic behavior is thus considered a rate process and is caused by the change of internal structure of material. One of its distinctive features to be depicted by a constitutive model seems to be the nonlinear rate sensitivity of inelastic flow. A tenfold change of loading rate causes a less than tenfold change in flow stress. The flow stress refers to the stress in the region of a stress-strain curve where the inelastic deformation is fully established and thus the tangent modulus is much smaller than the elastic modulus.

The loading rate sensitivity is classified into three different types; positive, zero and negative rate sensitivity. As normal deformation behavior, the positive and zero rate sensitivity stand for an increase and no change of the flow stress level with an increase in loading rate, respectively. On the other hand, the flow stress decreases with an increase in loading rate when the negative rate sensitivity is present. This negative sensitivity is considered to be a prerequisite for dynamic strain aging phenomenon, see Miller and Sherby (1978), Mulford and Kocks (1979) and Kubin and Estrin (1985). For the purpose of inelastic stress analysis in component design and material processing technology, one needs a constitutive model that is capable of depicting the diverse loading rate sensitivities in a consistent way.

An equilibrium state of a system represents no tendency to change with no change in the external control. But the existence of non-equilibrium state is an indispensable feature in understanding the hysteresis phenomenon of a stress-strain diagram and the rate-dependent behavior such as relaxation and creep. One of the feasible methodologies illuminating the irreversible dissipation processes is to introduce internal variables characterizing non-equilibrium conditions into the

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constitutive relations of a system, in addition to observable state variables on which the equilibrium state of a system depends uniquely. In this context, the current state of a system depends only on the current values of the observable variables and a set of internal variables. But the internal state variables do not appear explicitly in the expression of the energy conservation law of thermodynamics and are not explicitly coupled to any external control.

On account of usefulness of the internal state variables, the majority of plasticity and viscoplasticity models have adopted a way introducing the internal state variables so as to describe the history effect of the inelastic deformation behavior since Coleman and Noll (1959) and Coleman and Gurtin (1967) formalized the conceptual idea. The usage of the internal variables offers a means of capturing the overall effects of the complex interplay between internal structures and not the complicated causes at local state. As a natural consequence, the constitutive relations must be based on experimentally observed deformation behavior and be restricted by the laws of thermodynamics. The second law of thermodynamics, which takes the form of Clausius–Duhem inequality, serves as the main restriction in the formulation of constitutive relations. It is needed to postulate the existence of thermodynamic potential and dissipation potential in order to establish a complete set of constitutive relations that are consistent with the thermodynamics of irreversible processes.

Strain hardening and dynamic recovery are two competing mechanisms during inelastic deformation. The effective rate of strain hardening is thus reduced by the influence of dynamic recovery. The fundamental concepts to account for strain hardening are the formation of new dislocations and the interaction between individual dislocations when they pile up at a barrier such as grain boundary, precipitate and other dislocations. Dynamic recovery, which occurs simultaneously with deformation, is concerned with a reduction in the number of dislocations and the stored energy level. In most constitutive models for inelastic deformation behavior, the effect of two competing mechanisms is represented by the evolution law of the so-called back stress that is introduced in constitutive relations as an internal state variable. The evolution of the back stress has been based on the initial work of Armstrong and Frederick (1966) known as the nonlinear kinematic rule. Following the traditional approach of Lemaitre and Chaboche (1990) and Chaboche (1993, 1997), the present paper is to investigate the thermodynamic consistency of the constitutive model. This work is concentrated on the derivation procedure of a rate-dependent nonlinear kinematic rule, which is a fundamental ingredient in constitutive relations describing the three different types of loading rate sensitivity. The applicability of the constitutive model is also demonstrated by means of numerical simulations.

2. Constitutive model

A constitutive model is said to be thermodynamically consistent when the constitutive relations are derived in such a way that they satisfy the first and second law of thermodynamics. The first law is known as the principle of energy conservation. And the second law restricts the direction of energy transformation as an entropy principle. It is noted that there exist very strong mathematical similarities between the magnetization curve and the stress-strain curve; the stress and strain correspond to the magnetic field strength and magnetic flux density respectively. Thus the thermodynamic derivation procedure of the present work is akin to that of the magnetization model proposed by Ho (2014).

2.1. Thermodynamic framework

The principle of conservation of energy states that energy can be neither created nor be destroyed, but it can only be transformed from one form into another. For a closed thermodynamic system, the rate of the total energy is equal to the rate of work done on the system by the external forces plus the heat rate received by the system through heat flow across the system boundary and heat generation. On the assumption of small strain, the local form of the first law of thermodynamics is thus expressed as

$$\rho \dot{u} = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} + \rho \zeta - \nabla \cdot \mathbf{q} \tag{1}$$

where ρ is mass density, u the specific internal energy, $\boldsymbol{\sigma}$ the stress tensor, ε the strain tensor, ζ the internal heat source per unit mass, and $\mathbf{q}$ the heat flux vector. A superposed dot designates differentiation with respect to time. The colon represents the scalar product, $\boldsymbol{\sigma} : \boldsymbol{\varepsilon} = \sigma_{ij}\varepsilon_{ij}$ and the dot product of vectors $\mathbf{a}$ and $\mathbf{b}$ is defined as $\mathbf{a} \cdot \mathbf{b}$. A restriction on the direction in which the energy transformation process can be carried out in a system is given by the second law of thermodynamics. The second principle postulates that the rate of entropy production can never be negative:

$$\rho \dot{\eta} + \mathrm{div}\frac{\mathbf{q}}{T} - \rho \frac{\zeta}{T} \geq 0 \tag{2}$$

where η is the specific entropy and T is the temperature.

Rearranging the first law for ζ and substituting into Eq. (2), we obtain

$$\rho (T\dot{\eta} - \dot{u}) + \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} - \mathbf{q} \cdot \frac{\nabla T}{T} \geq 0 \tag{3}$$

This combination of the first and the second laws is referred to as the Clausius–Duhem inequality. Now we introduce the specific Helmholtz free energy as a thermodynamic state potential characterizing all thermodynamic properties of a material system. The Helmholtz free energy is related to the internal energy through the Legendre transformation:

$$\pi = u - T\eta \tag{4}$$