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Buckling analysis of arbitrary two-directional functionally graded Euler–Bernoulli nano-beams based on nonlocal elasticity theory

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ABSTRACT

Based on the nonlocal elasticity theory, buckling analysis of the nano-beams made of two-directional functionally graded materials (FGM) with small scale effects is carried out. To the best of the authors' knowledge, so far all previous solutions to the buckling analysis of arbitrary FGM Euler–Bernoulli nano-beams have addressed the case of properties varying in one direction only. The novelty of the current work is to present a solution by taking into account the variation of properties in two-directional functionally graded materials with arbitrary functions. The material properties obey the arbitrary function in thickness and length direction. The governing equations are obtained, employing the principle of minimum potential energy. Generalized differential quadrature method (GDQM) is selected in order to analyze the nonlocal beams with arbitrary boundary conditions along them to obtain the critical buckling load of FG nano-beam. These models can degenerate into the classical models if the material length scale parameter is taken to be zero. Finally, some numerical results are presented to study the effects of material length scale parameter and inhomogeneity constant on size dependent critical buckling load.

1. Introduction

Recently, nano and micro structural elements such as beams, membranes and plates have attracted worldwide attention from the research community for their superior properties and extensive applications in nano and micro electromechanical (NEM & MEMS) devices. At nano and micro meter scales, size effects often become important. Both experimental and Molecular dynamics simulation results have invariably shown that the small-scale effects in the analysis of mechanical properties of nano and micro structures cannot be neglected and that classical continuum theories cannot be applied. Molecular dynamics simulation is a convenient method to simulate the mechanical behavior of small size structures. However, it is computationally expensive for structures with large numbers of atoms (Daneshmehr, Rajabpoor, & Hadi, 2015). Thus researchers have been led to develop several higher-order continuum theories such as nonlocal theory (Eringen, 1972a, b, 1983, 2002), strain gradient theory (Lam, Yang, Chong, Wang, & Tong, 2003) and the like, which could predict size effect by considering material length scale parameters. Among the size dependent continuum theories, nonlocal elasticity theory initiated by Eringen and coworkers has been widely used to analyze many problems associated with nanostructures. According to the

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nonlocal elasticity theory, unlike classical elasticity, the stress tensor at an arbitrary point in the domain of material depends not only on the strain tensor at this point but also on strain tensor at all other points in the domain. Many researchers have applied the nonlocal elasticity concept for bending, buckling and vibration analyses of nano-sized structures such as beams (Aranda-Ruiz, Loya, & Fernández-Sáez, 2012; Aydogdu, 2009; de Sciarra, 2014; Ghannadpour, Mohammadi, & Fazilati, 2013; Loya, López-Puente, Zaera, & Fernández-Sáez, 2009; Lu, 2007; Miandoab, Pishkenari, Yousefi-Koma, & Hoorzad, 2014; Murmu & Pradhan, 2009a; Pradhan & Phadikar, 2009; Reddy & El-Borgi, 2014; Wang, Zhang, Ramesh, & Kitipornchai, 2006; Yang, Ke, & Kitipornchai, 2010), rods (Adhikari, Murmu, & McCarthy, 2014; Murmu & Adhikari, 2010; Murmu & Pradhan, 2009b) and plates (Ansari, Shahabodini, & Shojaei, 2016; Daneshmehr et al., 2015; Ke, Liu, & Wang, 2015; Zenkour & Sobhy, 2013; Zenkour, 2014).

Functionally graded materials (FGMs), whose spatially varying properties are tailored to satisfy particular engineering applications, have received considerable attention in recent years (Nejad & Fatehi, 2015). This continuously varying composition eliminates interface problems, and thus, the stress distributions are smooth (Kahrobaiyan, Rahaeifard, Tajalli, & Ahmadian, 2012). A number of papers considering various aspects of FGM have been published in recent years (Nejad, Rastgoo, & Hadi, 2014; M Şimşek & Reddy, 2013; Xue & Pan, 2013). Thanks to the advances in technology, FGMs have started to find their ways into micro/nanoelectromechanical systems (MEMS/NEMS), for example in the form of shape memory alloy thin films with a global thickness in micro- or nano-scale (Lü, Lim, & Chen, 2009b), electrically actuated MEMS devices (Zhang & Fu, 2012), and atomic force microscopes (AFMs) (Kahrobaiyan, Asghari, Rahaeifard, & Ahmadian, 2010). It should be noted that most of the above-mentioned analyses are related to FGMs with material properties varying in one direction only. However, there are practical occasions which require tailored grading of macroscopic properties in two or even three directions. As reported by Steinberg (1986), the fuselage of an aerospace craft undergoes an extremely high temperature field with excessive temperature gradient on the surface and through the thickness, when the plane sustains flight at a speed of Mach 8 and at an altitude of 29 km. In this circumstance, the conventional unidirectional FGMs may not be so appropriate to resist multi-directional severe variations of temperature. Therefore, it is of great significance to develop novel FGMs with macroscopic properties varying in two or three directions (2D or 3D FGMs) to withstand a more general temperature field (Lü, Chen, Xu, & Lim, 2008). The number of studies on beams and plates made of two-directional functionally graded material (2D-FGM) is still very limited. Lü et al. (2009a) proposed the state-space based differential quadrature method for the thermo-elastic analysis of bi-directional FGM plates. In addition, dynamic behavior of multi-directional FGM annular plates was investigated by Nie and Zhong (2010). Zhao, Chen and Lu (2012) suggested a symplectic framework for the analysis of plane problems of bi-directional functionally graded materials (FGMs), in which the elastic modulus varies exponentially both along the longitudinal and transverse coordinates. The fully coupled thermo-mechanical behavior of bi-directional functionally graded material (FGM) beam structures, using isogeometric finite element model was studied by Lezgy-Nazargah (2015). Şimşek (2015) investigated free and forced vibration of bi-directional functionally graded (BDFG) Timoshenko beam under the action of a moving load. The material properties of the beam varied exponentially in both axial and thickness directions. Wang et al. (2016) studied the free vibration of a two-directional functionally graded beam which had variable material properties along the beam length and thickness. It is assumed that material properties vary through the length according to a simple power law distribution with an arbitrary power index and have an exponential gradation along the beam thickness.

To the best of the researchers' knowledge, there are no studies carried out so far in the literature into two-directional functionally graded Euler–Bernoulli nano-beams for buckling analysis of nanostructures. Given this oversight, the aim of the present study is to investigate size effects on buckling analysis of two-directional functionally graded Euler–Bernoulli nano-beams. To accomplish this, the Eringen's nonlocal theory is applied to study the small scale effects. The material properties of the beam vary as arbitrary function in both axial and thickness directions. Finally, the size effect, and also the effect of nonlocal parameter and power index on critical buckling load are demonstrated with numerical results.

2. Analysis

Consider a nano-beam of length L, width b, and thickness h made of functionally graded materials (FGMs), as shown in Fig. 1. Cartesian coordinates (x, y, z) are considered.

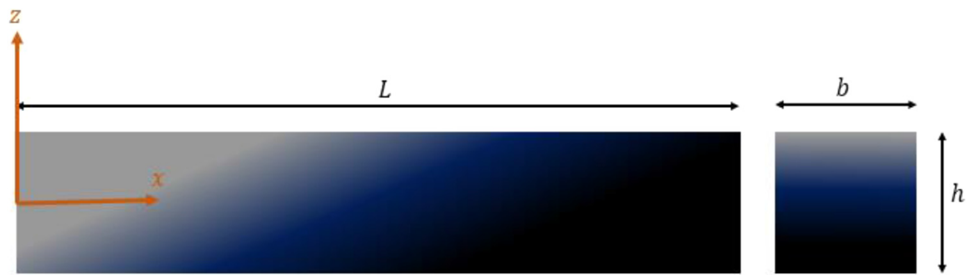

Fig. 1. Geometry of the two-directional functionally graded Euler–Bernoulli nano-beam.