International Journal of Engineering Science 103 (2016) 19–34

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Efficient homogenization techniques for elastic composites: Maxwell scheme vs. effective field method

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ARTICLE INFO

Article history:
Received 28 November 2015
Revised 16 March 2016
Accepted 16 March 2016
Available online 2 April 2016

Keywords:
Heterogeneous medium
Homogenization
Effective elastic properties
Maxwell scheme
Effective field method

ABSTRACT

The work is devoted to the comparative analysis of four homogenization techniques (the original and generalized Maxwell schemes and the one-particle and multi-particle effective field methods) in application to the problem of calculation of the effective elastic stiffness tensor of matrix composites. The generalized Maxwell scheme and multi-particle effective field method reduce the problem to calculation of elastic fields in a finite volume of the composite embedded in the infinite homogeneous matrix medium and subjected to a constant external stress or strain field. For the solution of this problem, an efficient numerical method based on Gaussian approximating functions and fast Fourier transform technique is used. The results of the methods are compared for composites with ellipsoidal inclusions which elastic moduli are much smaller or much larger than the moduli of the matrix. The case of hybrid composites with two different families of ellipsoidal inclusion is also considered. It is shown that for various shapes and elastic properties of inclusions, the generalized Maxwell scheme and multi-particle effective field method give close results. But in the case of hard inclusions of large volume fractions the results of these methods deviate substantially from the original Maxwell scheme that does not take into account interactions between inclusions.

1. Introduction

In his book “A Treatise on Electricity and Magnetism” (1873), James Clerk Maxwell proposed a method for calculation of effective resistivity of a homogeneous medium with an array of spherical particles (inclusions).

In this method, a sphere of the composite medium with a number of spherical particles was embedded in a homogeneous background medium. A sphere of the same radius made from the homogeneous material with the effective resistivity of the composite embedded in the background homogeneous medium was also considered (Fig. 1). Both media were subjected to the same constant external field (current).

The equation for the effective resistivity of the composite was derived from the condition that the far field asymptotics of the disturbed fields from the composite sphere and from the homogeneous ”effective” sphere coincide. In order to solve the problem, Maxwell assumed that each inclusion in the composite sphere is subjected to the external field applied to the medium. In other words, interactions between inclusions where neglected. It allowed solving the problem for each spherical particle and deriving an explicit equation for the effective resistivity that was called in the literature after Maxwell name.

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http://dx.doi.org/10.1016/j.ijengsci.2016.03.004
0020-7225/

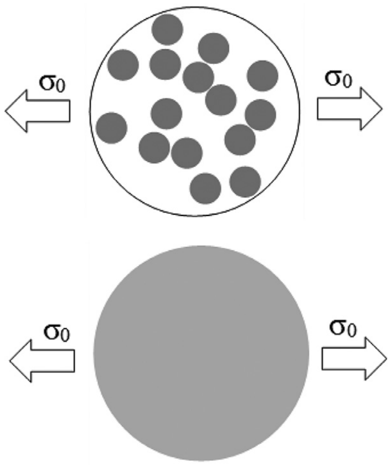

Fig. 1. The Maxwell homogenization method.

Maxwell himself recognized that his derivation was valid only for small volume fractions of inclusions. As R. Landauer wrote in his survey (Landauer, 1978): "… Later authors, however, have on occasion claimed more on behalf of Maxwell than is warranted".

Simplicity of the Maxwell scheme seemed to be attractive for many authors, and in a number of works the method was applied to calculation of effective conductive and elastic properties of composites with ellipsoidal inclusions and even hybrid composites containing different families of ellipsoidal inclusions. In turned out that in the case of elastic composites with spherical inclusions, the Maxwell scheme yields the same equations for the effective elastic constants as other homogenization methods that apparently take into account interactions between inclusions (e.g., the effective field method and Mori–Tanaka method (Berriman & Berge, 1996; Kanaun & Levin, 2013; Kuster & Toksõz, 1974)). So, there appeared an opinion that the results of the Maxwell scheme can be extended to the region of not small volume fractions of inclusions. Nevertheless one has to recognize evident drawbacks of the Maxwell scheme. It does not allow describing influence of structure in the inclusion positions on the effective properties. Its predictions strongly depend on the shape of the composite volume considered in the Maxwell scheme (for spherical and non-spherical (ellipsoidal) shapes of this volume, the method predicts different effective properties of the composite Berriman and Berge, 1996). Such a dependence gives rise the question about the shape of the composite volume in the Maxwell scheme that corresponds to the given composite structure. But in the framework of the scheme, this question cannot be answered.

In Kanaun and Babaii Kochecksaraii (2008), Kanaun (2011b) an efficient numerical method for solution of the electrostatic and elasticity problems for a homogeneous medium with several heterogeneous inclusions was developed. In Kanaun (2010), Kanaun (2011a) this method was used in the framework of the multi-particle effective field method for calculation of the effective conductivity and elastic stiffness of matrix composites. It was shown that the method predictions coincide with the result of other numerical and analytical methods of calculation of effective conductive and elastic properties of regular and random composites. The method opens the way for generalization of the Maxwell scheme by numerical solution of the problem for the composite sphere embedded in the infinite matrix medium. If this problem can be solved numerically, interactions between inclusions would be taken into account automatically. The same idea based on another numerical technique was used in Mogilevskaya, Kushch, Koroteeva, and Crouch (2012) for calculation of effective conductivity of regular composites.

In this work, the original Maxwell scheme is combined with numerical solution of the elasticity problem for several isolated inclusions in an infinite background medium. Farther, this method of homogenization is called the generalized Maxwell scheme.

The case of a composite that consists of a set of grains randomly distributed over orientations is considered. In each grain, ellipsoidal inclusions are arranged in a face centered (FC) lattice. Elastic moduli of the inclusion material are assumed to be much smaller or much larger than the moduli of the matrix phase. The case of a hybrid composite with two different families of ellipsoidal inclusions is also considered. The results of the original and generalized Maxwell schemes are compared with the predictions of the one-particle and multi-particle effective field methods.

2. Integral equations of elasticity for composite media

Let an infinite homogeneous medium with the elastic stiffness tensor $\mathbf{C}^0$ contain an array of isolated inclusions that occupy regions v^k $(k = 1, 2, \ldots)$ with elastic stiffness tensors $\mathbf{C}^k$. The medium is subjected to a constant external strain field $\boldsymbol{\varepsilon}^0$ (stress field $\boldsymbol{\sigma}^0$), and the objective is to calculate strain and stress fields in the medium with the inclusions and assess the effective elastic stiffness tensor of the composite. The problem can be reduced to volume integral equations for the strain $\boldsymbol{\varepsilon}(x)$ or stress $\boldsymbol{\sigma}(x)$ tensors in the heterogeneous medium. Here $x(x_1, x_2, x_3)$ is a point of 3D-space, and $\mathbf{x}$ is the vector of