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A note on the steady flow of Newtonian fluids with pressure dependent viscosity in a rectangular duct

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ABSTRACT

Unidirectional steady flow of Newtonian liquids with a pressure-dependent viscosity in a rectangular duct is considered. Governing momentum equation is reduced to a quasilinear second order elliptic partial differential equation. We give an analytical solution to the governing equation, and investigate the effect of aspect ratio and pressure coefficient on the velocity profiles numerically.

1. Introduction

The viscosity of fluids, such as polymer melts and lubricants, depends strongly on temperature and to a lesser extent on pressure (Rajagopal, 2006). In some cases the dependence of the viscosity on pressure may be several orders of magnitude stronger than that of density (Rajagopal, 2006; Stokes, 1845). Stokes, in his famous paper (Stokes, 1845), discusses the possibility that the viscosity of a fluid may vary with the pressure. Barus (1983), and later on Bird, Amstrong, and Hassager (1977) (see also the book by Bridgman, 1931) experimentally showed that viscosity grows exponentially with increasing pressure. Further details and references to more recent experimental studies can be found in the book by Szeri (1998) and in the paper by Málek and Rajagopal (2007) that reflects the situation before 2006. Recent papers by Bair and Kottke (2003) and by Bair (2006) report even drastically faster dependence of the viscosity on the pressure. It should be noted that even at such higher pressures the variation in the density of most liquids, in comparison with the variations in the viscosity, is negligible, as discussed in Rajagopal (2006) or Malek and Rajagopal (2007). As a consequence, these liquids can be modelled as incompressible materials.

Mathematical issues arising in the case of incompressible Newtonian or non-Newtonian flows with a pressure-dependent viscosity have been addressed by Renardy (2003), Gazzola (1997), Hron, Málek, Něcas, and Rajagopal (2003) and Málek, Něcas, and Rajagopal (2002). The existence of flows of fluids with pressure dependent viscosity and the associated assumptions have been discussed by Bulíček, Málek, and Rajagopal (2008). The properties of such solutions have been investigated by Málek and Rajagopal (2007).

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Hron, Málek, and Rajagopal (2001) were the first authors to show that steady unidirectional flow is possible if the dependence of the viscosity on the pressure is linear even if shear-thinning effects are included. However, they also show that for other forms of dependence of the viscosity on pressure, such as polynomial and exponential dependence, unidirectional flow is not possible. In the case of planar flows, a pressure driven parallel flow exists only if the dependence of viscosity on pressure is linear (Renardy, 2003). It turns out that in the case of linear dependence actually parallel flow always exists regardless of the cross-section of the pipe. Denn (1981) showed that the quadratic velocity profile in a circular pipe remains a solution if the viscosity is an exponential function of the pressure. As indicated by Renardy (2003) and also shown in the present work, the velocity profile is not parabolic in the case of linear dependence of the viscosity; it may be almost parabolic when this dependence is weak. According to Suslov and Tran (2008), the major concern of linear dependence is that it does not guarantee positive definiteness of the viscosity which requires the pressure to remain positive. This problem is not encountered when using exponential dependence or in flows where the pressure remains positive, such as Poiseuille flows. More recently, Kalogirou, Poyiadji, and Georgiou (2011) obtained the analytical solution of axisymmetric, annular, and plane Poiseuille flows of Newtonian fluids with pressure-dependent viscosity with a linear dependence on viscosity,

$$\eta(p) = \eta_0(1 + \beta p) \tag{1.1}$$

In this work, we consider laminar flow in a straight duct of rectangular cross-section of Newtonian fluids with pressure-dependent viscosity defined in (1.1). The rest of the paper is organized as follows: in Section 2 the governing equation, a quasilinear second order elliptic partial differential equation, derived from the linear momentum equation is presented and novel semi analytical results are obtained. In Section 3, the results including the effects of the viscosity pressure-dependence is discussed.

2. Governing equations

We shall consider flow of an incompressible fluids whose Cauchy stress $\boldsymbol{T}$ is given by

$$\boldsymbol{T} = -p\boldsymbol{I} + 2\eta(p)\boldsymbol{D} \tag{2.1}$$

where

$$\boldsymbol{D} = \frac{1}{2}\left[\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T\right] \tag{2.2}$$

$\boldsymbol{D}$ is the rate-of-deformation tensor and $\boldsymbol{u}$ is the velocity vector. Then governing momentum equations becomes

$$\rho\left(\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{x}} + \boldsymbol{u}.\ \nabla\boldsymbol{u}\right) = -\nabla p + \eta(p)\nabla^2\boldsymbol{u} + 2\eta'(p)\nabla p.\boldsymbol{D} \tag{2.3}$$

We consider here flow of a fluid modelled by (2.1) in a straight duct with rectangular cross-section due to the prescribed values of the pressure at two different places along the duct or prescribed total flux and pressure value at one point. Flow is fully developed and unidirectional in the axial z-direction. The velocity vector is given by $\boldsymbol{u} = [0, 0, w(x, y)]$. Then the governing Eq. (2.3) reduces to,

$$-\frac{\partial p}{\partial x} + \eta_0\beta\frac{\partial p}{\partial z}\frac{\partial w}{\partial x} = 0 \tag{2.4}$$

$$-\frac{\partial p}{\partial y} + \eta_0\beta\frac{\partial p}{\partial z}\frac{\partial w}{\partial y} = 0 \tag{2.5}$$

$$-\frac{\partial p}{\partial z} + \eta_0\beta\frac{\partial p}{\partial x}\frac{\partial w}{\partial x} + \eta_0\beta\frac{\partial p}{\partial y}\frac{\partial w}{\partial y} + \eta_0(1+\beta p)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = 0 \tag{2.6}$$

Substitution of (2.4) and (2.5) into (2.6) yields,

$$-\frac{\partial p}{\partial z}\left(1 - (\eta_0\beta)^2\left(\frac{\partial w}{\partial x}\right)^2 - (\eta_0\beta)^2\left(\frac{\partial w}{\partial y}\right)^2\right) + \eta_0(1+\beta p)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = 0 \tag{2.7}$$

Governing field equations are rendered dimensionless using for scale factors the width L and the height H of the rectangular cross-section, the mean velocity U, $3\eta_0 LU/H^2$ and the zero-pressure viscosity η_0 for the transversal coordinates x, y, velocity $w(x, y)$, pressure p and the viscosity η, respectively. The resulting dimensionless viscosity, the counterpart of (1.1) becomes,

$$\eta^* = 1 + \varepsilon p^*,\ \ \varepsilon = \frac{3\beta\eta_0 U}{H}, \tag{2.8}$$