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On the overall yielding of an isotropic porous material with a matrix obeying a non-quadratic criterion

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ABSTRACT

Gurson's approach for investigating the overall response of a rigid-plastic porous representative volume element (RVE) leads to explicit upper-bound estimates only in few particular cases, e.g., when the matrix of the RVE obeys a quadratic criterion (von Mises or Hill'48). The formal difficulties that prevent the application of Gurson's methodology to a wider range of materials, of current technological interest, can be circumvented by a numerical approach. We show illustrations for an idealized RVE in the form of a hollow sphere with von Mises and Hershey-Hosford matrix, respectively. In both cases the overall yield surface is revealed to have a complex geometry, the most notable features being the variation of the overall measure of equivalent stress along the pressure axis and a significant asymmetry of its level sets in the range of high triaxialities.

1. Introduction

Modeling the plastic deformation of metals while taking into account the gradual deterioration of their load carrying capacity due to the nucleation, growth and coalescence of voids (or pores) at microstructural level has been and still is an active field of research, motivated by important practical applications.

By contrast with the "classical" incompressible plastic behavior[1], a distinctive characteristic of porous metals, manifesting at the macroscopic level of observation, is that they feature a significant volumetric component of plastic deformation, or, equivalently, that their stress-strain response is influenced by the hydrostatic component of the stress state. This is explained by the fact that, even if the macroscopically applied stress is purely hydrostatic, the stress field generated at microscopic level has in general a non-vanishing deviatoric component, being thus capable of plastically deforming a plastically incompressible matrix (sound material).

The first isotropic yield function inspired by the just described micro-mechanism was proposed by Gurson (1977) in the form

$$\mathcal{F}(\boldsymbol{\Sigma}, f, \overline{\epsilon}_p) = \frac{\Sigma_{eq}^2}{H^2} + 2f\cosh\left(\frac{3\Sigma_m}{2H}\right) - 1 - f^2 \tag{1}$$

where $\boldsymbol{\Sigma}$ denotes the macroscopic (or, from an averaging perspective, the overall) Cauchy stress, $\Sigma_{\mathrm{m}} := \mathrm{tr}\boldsymbol{\Sigma}/3$ its hydrostatic component, $\boldsymbol{\Sigma}' := \boldsymbol{\Sigma} - \Sigma_m \boldsymbol{I}$ its deviatoric part, and $\Sigma_{eq} := \sqrt{3/2}|\boldsymbol{\Sigma}'|$ its von Mises norm; the parameter $\overline{\epsilon}_p$ represents a

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[1] Where the volumetric component of deformation is elastic and hence negligible.

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macroscopic equivalent plastic strain, with $H = H(\overline{\epsilon}_p)$ representing isotropic hardening, while f is a measure of the porosity of the material.

The expression in Eq. (1) was deduced by Gurson based on the analysis of the rigid-plastic response of a representative volume element (RVE) idealized in the form of a hollow sphere with matrix obeying the von Mises yielding criterion. However, put in a wider perspective, formula (1) appears to be a fortunate event: Indeed, further extensions of Gurson's analysis, which are amenable to explicit calculations, seem possible only when the matrix is governed by a quadratic yielding criterion, von Mises or Hill'48, e.g., see Gologanu, Leblond, and Devaux (1994), Mariani and Corigliano (2001), for extensions to non-spherical void shapes, e.g., Trillat and Pastor (2005), Monchiet, Charkaluk, and Kondo (2011), for further refinements of Gurson's criterion, e.g., Leblond, Perrin, and Devaux (1995) and Pardoen and Hutchinson (2000) for extensions to hardenable matrix, and also Liao, Pan, and Tang (1997) and Benzerga and Besson (2001) for extensions to anisotropy of the matrix. While adequate for certain steels, this description of matrix yielding is less suited, for example, for non-ferrous metals such as aluminum or magnesium alloys. Then a more realistic modeling of the progress of damage by void growth, in a wider range of materials, requires the consideration of more general descriptions of matrix yielding behavior.

In this context, by comparison with the von Mises function, the Hershey (1954) and Hosford (1972) function[2]

$$g(\boldsymbol{\sigma}) = k_n[(\sigma_1 - \sigma_2)^n + (\sigma_2 - \sigma_3)^n + (\sigma_3 - \sigma_1)^n]^{1/n} \tag{2}$$

with exponents $n = 6$ or 8, is known to be closer to the experimental data of many materials with face centred cubic lattice, particularly aluminum. In addition, this function is at the origin of several anisotropic extensions (by linear transformations) that are specifically designed for the constitutive modeling of many materials of current technological interest, e.g., Karafillis and Boyce (1993), Barlat et al. (2005), Soare and Benzerga (2016).

It is then desirable to adapt Gurson's approach such that the study of an RVE with matrix obeying any given yielding criterion becomes feasible. A first logical step consists in treating the isotropic case. This is the primary objective here. As methodology, we adopt the same framework as in Gurson (1977)[3], but, instead of pursuing analytical calculations, we look for best approximations within an appropriate set of functions. As illustration, we present an estimate of the overall yield function of an RVE idealized as a hollow sphere whose matrix is rigid, perfectly plastic and governed by the Hershey–Hosford yield function with exponent $n = 8$.

At this point it must be mentioned that other approaches, some extending the Hashin-Shtrikman variational lemma for elastic bodies to nonlinear stress-strain relationships in potential form, e.g., Castaneda (1991), Willis (1991), or employing the Mori-Tanaka averaging scheme to estimate the elastic stress field in the presence of pores, Sevostianov and Kachanov (2001), while others adopting right from the outset an interpolative approach, e.g., Cocks (1989), Michel and Suquet (1992), describe the overall response of an isotropic porous material using a homogeneous yield function of the form

$$\mathcal{F}(\boldsymbol{\Sigma}, f, \bar{e}_p) = \left[A(f)\Sigma_{eq}^2 + B(f)\Sigma_m^2\right]^{1/2} - H(\bar{e}_p) \tag{3}$$

The parameters A and B are specific to each approach, but it may already be noted that they can be linked to purely deviatoric and hydrostatic responses at macro-level, respectively. In general, the above quadratic provides a satisfactory description only for large porosity ratios, e.g., $f \geq 0.01$, its performance being quite poor for lower porosities, the case of most interest in practice. This suggests that higher order polynomial combinations could be used for an improved description.

Then the course of this investigation is as follow. In Section 2 we review the general homogenization framework and outline the identification procedure for the overall yield function; in Section 3.1 we describe the numerical approach for the investigation of the overall yield surface of an RVE with matrix governed by an arbitrary isotropic yield function; applications to the cases $n = 2$ and $n = 8$ in Eq. (2) are featured in Section 3.2 and 3.3, respectively; finally, conclusions are drawn in Section 4.

2. Constitutive framework, averaging considerations and preliminary remarks

The object of study here is a representative volume element (RVE) of a macroscopic body, the overall response of the RVE defining the constitutive response at a particle X of the macroscopic body. The RVE occupies a domain Ω, with a subset $\omega \subset \Omega$ occupied by voids, the sound material, or matrix, occupying the set $\Omega \backslash \omega$. The porosity of the RVE is then the ratio $f := |\omega|/|\Omega|$.

We seek the response of the RVE when the macroscopic rate of deformation $\boldsymbol{D}$ at particle X is prescribed. Then the fundamental homogenization assumption is

$$\boldsymbol{v}(x, t) = \boldsymbol{D}(t) : x, \quad x \in \partial\Omega \tag{4}$$

where x denotes positions of particles of the RVE with respect to some laboratory frame, $\boldsymbol{v}$ is the velocity field within the RVE and $\partial\Omega$ is the (outer) boundary of the set Ω.

[2] σ_i are the principal values of the stress state $\boldsymbol{\sigma}$ and $k_n = 2^{-1/n}$ is a normalization constant chosen such that the yielding criterion $g(\boldsymbol{\sigma}) = h$ reduces to $\tau = h$ for uniaxial stress states of magnitude τ.

[3] See also Leblond (2003) for a modern exposition; however, for completeness, the necessary background is reviewed here in Section 2.