International Journal of Engineering Science 104 (2016) 20–33

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

A simple method for solving adhesive and non-adhesive axisymmetric contact problems of elastically graded materials

Markus Heß*

Institute of Mechanics, Technical University of Berlin, Berlin, 10623, Germany

ARTICLE INFO

Article history:
Received 14 March 2016
Revised 6 April 2016
Accepted 13 April 2016
Available online 22 April 2016

Keywords:
Functionally graded materials
Power-law graded half-space
Normal contact with adhesion
Winkler foundation
Method of dimensionality reduction
JKR-theory

ABSTRACT

An efficient method is presented for solving axisymmetric, frictionless contact problems between a rigid punch and an elastically non-homogeneous, power-law graded half-space. Provided that the contact area is simply-connected profiles of arbitrary shape can be considered. Moreover, adhesion in the framework of the generalized JKR-theory can be taken into account. All results agree exactly with those given by three-dimensional contact theories. The method uses the fact that three-dimensional contact problems can be mapped to one-dimensional ones with a properly defined Winkler foundation; hence, the method is to be understood as an extension of the method of dimensionality reduction (MDR). A prerequisite of its applicability forms the generality of contact stiffness regardless of the geometry of the axisymmetric profile, which is proved. All the necessary mapping rules are derived and their ease of use explained by a recent example.

1. Introduction

1.1. Fundamentals of functionally graded materials

The technological progress and the demand for increasing the capabilities require the development of high-performance materials. This class includes the functionally graded materials (FGMs), whose application field ranges from biomechanics, tribology and optoelectronics to nanotechnology (Miyamoto, Kaysser, Rabin, Kawasaki, & Ford, 1999). Unlike homogeneous materials, the material properties of FGMs can be adjusted optimally and partially independent from each other. In this way, high corrosion, fatigue and wear resistance, high fracture toughness and improved thermal and electrical properties can be combined in one material (Suresh, 2001). In engineering, such materials are used in order to withstand the enormous tribological stresses and thus to increase the reliability and service life of mechanical components.

Due to their optimized properties FGMs are associated with considerable cost. Therefore, mathematical models are needed as a basis of numerical simulations to predict the behavior of FGMs. In this context, a homogenization of the heterogeneous microstructure of the material occurs. FGMs are idealized as continua with spatially varying mechanical properties to enable the application of the continuum theory (Jha, Kant, & Singh, 2013). The spatial variation of the material properties perpendicular to the surface is usually described by an exponential or a power law.

However, contact problems of graded materials did not have to be re-investigated. Long before one spoke about FGMs, calculations in geomechanics have been done that studied the influence of an increasing elastic modulus with depth on the

* Tel.: +49 030 314 21485; fax: +49 (0)30/314-72575.
E-mail address: markus.hess@tu-berlin.de

http://dx.doi.org/10.1016/j.ijengsci.2016.04.009

settlements of soil (see e.g. Borowicka, 1943 ; Fröhlich, 1934). Over the years, various functions for the change of modulus were adopted. Good overviews can be found in the works of Selvadurai (2007) and Aleynikov (2010). In most cases the calculations are very complicated and usually only numerical solutions are possible. The majority of articles deal with an exponential or power-law dependent increase of the modulus of elasticity. Moreover, almost all presuppose a frictionless normal contact. The occurrence of tangential stresses on the surface or the taking into account of adhesion complicates a problem solution significantly. Due to the rapid development in nanotechnology the investigation of adhesive contact problems between FGMs came into focus. Although non-adhesive contact problems of elastically non-homogeneous, power-law graded materials of different shape have been studied for a long time (Booker, Balaam, & Davis, 1985a; Giannakopoulos & Suresh, 1997), contact problems with adhesion are the subject of current research (Chen, Yan, Zhang, & Gao, 2009; Jin, Guo, & Zhang, 2013; Jin, Zhang, Wan, & Guo, 2015). In this paper we present an alternative, highly efficient method that leads to exact solutions of the above-mentioned contact problems. Due to its simplicity it offers any budding engineer access to the solution of adhesive contact problems involving graded materials. This requires only the basic knowledge of analysis and numerical simulation. The method is known as *method of dimensionality reduction* (MDR).

1.2. The method of dimensionality reduction

Within tribological systems the solution of contact problems can be very difficult even for homogeneous, elastic or viscoelastic materials. On the one hand it is necessary to distinguish between normal, tangential and rolling contact, on the other hand, adhesion, thermal and electrical effects and lubricant or foreign layers must be taken into account (Popov, 2015b). Whereas the contact behavior of single contacts is affected by the contact profile, in a rough contact the multi-scale nature of the roughness has to be considered. This is just a typical range of problems, which reveals the need for numerical simulations in order to make reliable statements about the friction and wear behavior or on the electrical and thermal resistance. Due to the variety and complexity of contact problems their solution is only reserved for a small research-based group. The solution requires the knowledge of very different three-dimensional theories: Hertz's theory of normal contact (1882), theory of tangential contact of Cattaneo (1938) and Mindlin (1949), theorem of Ciavarella (1998) and Jäger (1995), the JKR-theory of adhesive contact (Johnson, Kendall, & Roberts, 1971) or the theory of Radok (1957) for the description of viscoelastic material behavior, to name just a few. This is just one of the reasons why the method of dimensionality reduction (MDR) has been developed. MDR makes all theories for solving three-dimensional contact problems exactly to one-dimensional theories. Simply speaking, the contact between two three-dimensional elastic half-spaces is mapped on to the contact between a rigid, plane indenter and a one-dimensional series of springs.

After Popov (2005) provided the basic idea of MDR, mainly the works of Geike and Popov (2007), Heß (2011) and Pohrt, Popov, and Filippov (2012) have contributed to its development. MDR has been continuously expanded, so that it is now applied to the solution of tangential and rolling contact problems between transversely isotropic and viscoelastic materials, or for the calculation of electric and thermal resistances. Here we do not want to list all the associated papers, but refer to the comprehensive and latest work of Popov and Heß (2015). It should be mentioned that even reliable wear calculations (Dimaki et al., 2015; Popov, 2014) as well as calculations of torsional contacts (Willert, Heß, & Popov, 2015) can be done by MDR. We would like to point out, that the applicability of MDR to contact problems between randomly rough surfaces is controversially discussed (Persson, 2015; Popov, 2015a). However, its applicability to the solution of axisymmetric contact problems with a simply-connected contact area is no question and the associated field of application is large. Applications within engineering range from nano- and micro-electromechanical systems (NEMS, MEMS), bearings, joints, couplings and gears up to the wheel-rail contact. In addition, MDR provides the exact solution of the tangential contact between rough surfaces if the solution of the normal contact is given (Popov, Pohrt, & Heß, 2015). In a recent paper Argatov (2015) discusses the advantages and disadvantages of MDR. A summary of all the mapping rules of MDR related to the solution of axisymmetric contact problems can be found in a paper by Popov and Heß (2014).

All the above mentioned works refer to the contact of elastic homogeneous half-spaces. The applicability of MDR to heterogeneous media is discussed only in one paper by Popov (2014a). Therein, Popov clarifies in detail all essential characteristics of axisymmetric contact problems between heterogeneous media, which enable a reduction. Exact and approximate solutions using MDR are compared and applied to contact problems of coated solids. This paper is dedicated to the application of MDR on contact problems with inhomogeneous elastic materials. We assume that the elastic modulus increases with depth perpendicular to the surface of the half-space according to the power-law

$$E(z) = E_0\left(\frac{z}{c_0}\right)^k \quad \text{with} \quad 0 \le k < 1. \tag{1}$$

A simply connected contact area and a constant Poisson's ratio in the range of $0 \le \nu \le 0.5$ are assumed. Although it is known that a negative Poisson's ratio significantly affects the indentation analysis (Argatov, Guinovart-Díaz, & Sabina, 2002) the investigation of auxetic materials should not be a subject of this paper.

Due to the vanishing modulus at the surface Eq. (1) must be understood as an idealization. Nevertheless, this approach is justified. FEM calculations by Lee, Barber, and Thouless (2009) have shown that the principal contact behavior is not very different in comparison with a more realistic, piecewise defined law. As mentioned at the beginning, many FGMs exhibit such behavior. We will prove that any axisymmetric contact problem involving such FGMs and including adhesion can be solved exactly by using MDR.