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Thermo-electro-elastic analysis of functionally graded piezoelectric shells of revolution: Governing equations and solutions for some simple cases

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A B S T R A C T

In this article, the three-dimensional multi-field equations of functionally graded piezoelectric (FGP) shells of revolution under thermo-mechanical loading are derived. First, the heat conduction equation for an FGP is derived and then, the displacement equations are developed considering thermal effects. The coupling is one-way, i.e. the temperature field affects the displacements and stresses while the back influence of the displacement on the temperature is disregarded. The Hamilton's principle is used to derive the governing equations of motion, in presence of system rotation effects, for thick shells of revolution with variable thickness and arbitrary curvature. Material properties are assumed to vary in various directions according to an arbitrary function. For the sake of simplicity and verification of derivation, the heat conduction and governing equations of motion have been reduced for a functionally graded piezoelectric cylindrical shell under thermal loading. In order for the general equations to be verified, two simple examples are investigated, i.e. thermal stresses in hollow cylinder and sphere. Correctness and generality of the present results can be justified given the capability of these equations for different geometries and material properties.

1. Introduction

There is a tremendous demand on the simultaneous mechanical, thermal and chemical resistance of structural elements in advanced engineering technologies. There is a great potential of fulfilling such demands by using functionally graded materials (FGM) (Sofiyev, 2016). FGM is an advanced class of heterogeneous composite materials where mechanical properties vary smoothly and continuously from one point to the other (Mohammadi, Saha, & Akbarzadeh, 2016; Ninh & Bich, 2016; Salehipour, Sahidi, & Nahvi, 2015). The material composition can be designed so as to improve the strength, toughness, high-temperature withstanding ability, etc., to meet the desired structural performance in different engineering applications (Ke, Yang, Kitipornchai, & Wang, 2014). Functional grading of material can be used to achieve a variety of goals, including alleviation of residual stresses, reducing stresses during lifetime of the structure, improvement of stability and dynamic response, preventing fracture and fatigue, etc. (Birman, 2014). There are many studies in the literature carried out into the analysis of functionally graded (FG) structures with various solution methods (Fatehi & Nejad, 2014; Jabbari, Nejad, & Ghannad, 2015;

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Moosaie, 2016; Nejad & Fatehi, 2015; Nejad, Jabbari, & Ghannad, 2015; Nejad & Kashkoli, 2014; Tokovyy & Chien-Ching, 2015; Wu & Wang, 2015).

Piezoelectric materials have been widely used as sensors and actuators in control systems due to their excellent electromechanical properties, design flexibility, and efficiency to convert electrical energy into mechanical energy or vice versa (Li & Pan, 2015). In order to overcome the performance limitations of the traditional layered piezoelectric elements, the concept of FGM has been extended into the piezoelectric materials by recent advances in the metallurgical science and fabrication techniques (Li & Pan, 2015).

Over the past years, some researchers have managed to analyze thick shells of revolution for isotropic and functionally graded materials. Semi-analytical finite element formulation, using first-order shear deformation theory, was used by Ganesan and Kadoli (2005) to analyze the shell of revolution in curvilinear coordinate. Kang (2007) employed the three-dimensional formulation of elasticity for modeling a thick shell of revolution with variable thickness and curvature. The achieved formulation in that study was valid for an isotropic material. Nejad, Rahimi, and Ghannad (2009) could derive a set of field equations for a functionally graded shell of revolution. The tensor-based formulation was employed for the purpose of analyzing a functionally graded piezoelectric shell of revolution by Arefi and Rahimi (2012). Arciniega and Reddy (2007a, 2007b) suggested the finite element formulation for nonlinear analysis of a shell structures based on tensor analysis. A curvilinear coordinate system with higher-order elements has been employed for this purpose.

There are several researches in the field of thermal investigation in conjunction with electro-elastic analysis of shells. Ootao (2009), Ootao and Tanigawa (2007), Ootao, Akai, and Tanigawa (2008) widely investigated transient thermo-electro-elastic analysis of the spherical and cylindrical shells. Khoshgoftar, Arani, and Arefi (2009) suggested an analytical approach to thermo-elastic analysis of a thick-walled cylinder made of functionally graded piezoelectric material. Sheng and Wang (2010) reported on the analysis of thermo-elastic vibration and buckling characteristics of a functionally graded piezoelectric cylinder. The numerical differential quadrature (DQ) method was employed for three-dimensional nonlinear thermo-elastic analysis of a functionally graded cylindrical shell with piezoelectric layers by Alashti and Khorsand (2012). Lang and Xuewu (2013) considered the coupling effect between mechanical, electrical, magnetic and thermal loadings, using Hamilton's principle and higher-order shear deformation theory. So far, thermo-elastic analysis of functionally graded piezoelectric shells with common geometries, such as cylinders and spheres, have been extensively performed in the literatures (Alashti, Khorsand, & Tarahhomi, 2013; Dai, Hong, Fu, & Xiao, 2010; Dai & Rao, 2011; Gharooni, Ghannad, & Nejad, 2016; Jabbari, Karampour, & Eslami, 2013; Liu, Bian, Chen, & Lü, 2012; Mazarei, Nejad, & Hadi, 2016; Saadatfar & Razavi, 2009).

The main purpose of the present study is to develop the general form of equations in coupled electro-thermo-elastic analysis of shells of revolution with variable thickness and material properties that are graded in three directions. In this paper, primarily, the basic equations in a curvilinear system are introduced and base vectors, in the form of covariant and contra-variant, are developed. Equations of heat conduction in the current curvilinear coordinate system have been derived. This type of equations can be easily transformed to cylindrical, spherical or any existing ordinary orthogonal curvilinear coordinates systems. The constitutive relations for coupled electro-thermo-elastic behavior of material are developed. The energy method and Hamilton's principle are used in order to obtain energy functional and derive the equations of motion.

2. Problem formulation

2.1. Basic equations in a curvilinear coordinate system

The thermal analysis of a functionally graded shell in a curvilinear coordinate system is presented here using tensor analysis. For this purpose, a relationship is written between original Cartesian and curvilinear coordinate systems. Primarily, a surface of revolution containing a plane curve is considered to be rotating around an axis in its plane. This curve is technically called a meridian. Variables that indicate components of the curvilinear coordinate system are $(\varphi, \Omega, \theta)$, which are meridian, normal distance and circumferential components, respectively. As shown in Fig. 1 for the meridian of the middle surface of a shell, R represents the distance of an arbitrary point, located in the middle curve, from the axis of rotation x-axis; R_1 and R_2 are the two principal radii of curvature.

Using Fig. 1, the following relations can be defined

$$R = R_2 \sin\varphi \tag{1}$$

$$\mathrm{d}s = R_1 d\varphi \tag{2}$$

$$\mathrm{d}R = \mathrm{d}s \cos\varphi \tag{3}$$

$$\mathrm{d}x = \mathrm{d}s \sin\varphi \tag{4}$$