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Small-scale indentation of an elastic coated half-space: The effect of compliant substrate

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ABSTRACT

Under consideration in this paper is the axisymmetric problem of unilateral frictionless indentation of a thin stiff film bonded to a compliant substrate, which is assumed to be a homogeneous isotropic elastic half-space. The previously developed first-order asymptotic model for the small-scale indentation, when the contact radius is small compared to the film thickness, is compared with approximate solutions based on the Kirchhoff thin plate theory with satisfactory results.

1. Introduction

Indentation testing has emerged as one of indispensable tools for mechanical characterization of thin elastic layers (films, coatings) deposited on elastic substrates (Antunes, Fernandes, Sakharova, Oliveira, & Menezes, 2007; Chen & Vlassak, 2001). The corresponding indentation problems have been the subject of many investigations over a number of years (Argatov, 2010; Gao, Chiu, & Lee, 1992; Perriot & Barthel, 2004).

The influence of the elastic substrate on the indentation data is usually called the substrate effect (Argatov & Sabina, 2013; Epshtein, Borodich, & Bull, 2015; Huang & Chang, 2010; Xu & Pharr, 2006). In the case of a relatively compliant coating, the substrate can be assumed to be absolutely rigid, and the substrate effect reduces to the so-called thickness effect (Argatov, Daniels, Mishuris, Ronken, & Wirz, 2013; Argatov, 2011; Argatov, Guinovart-Díaz, & Sabina, 2012; Doerner & Nix, 1986; Hayes, Keer, Herrmann, & Mockros, 1972), which depends on both the layer thickness and the boundary condition at the layer/substrate interface.

It is known (Lee, Barber, & Thouless, 2009) that in the case of a relatively compliant substrate (that is in the case of a relatively soft coating), the small-scale indentation response, h, of a relatively stiff coating film deposited on the substrate represents not only the local contact deformation beneath the indenter, but also the global deformation, h_1, of the elastic film/substrate system (see Fig. 1). As a first approximation, the latter can be modeled in the framework of the bending theory for an infinite elastic Kirchhoff plate (of thickness t and bending stiffness $D_f = t^3 E_f/[12(1-\nu_f^2)]$ with E_f and ν_f being Young's modulus and Poisson's ratio of the film material) deposited on an elastic half-space (with Young's modulus E_s

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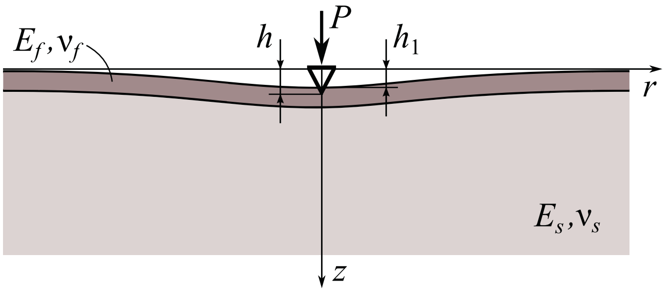

Fig. 1. Schematics of the small-scale indentation test for a stiff film deposited on a compliant elastic substrate. A rigid indenter achieves the indentation displacement h, a part of which, h_1, can be attributed to the global bending deformation of the stiff film.

and Poisson's ratio ν_s) and loaded by a concentrated force, P. The same notation is followed here as in (Argatov & Sabina, 2014).

In the situation when the interface between the infinite elastic plate and the compliant elastic half-space (substrate) is frictionless, the solution to the bending problem was given by Timoshenko and Woinowsky-Krieger (1969) in the form

$$w(r) = \frac{Pl^2}{2\pi D_f} \int_0^\infty \frac{J_0(\lambda r/l)\,d\lambda}{1+\lambda^3}, \tag{1}$$

where $J_0(x)$ is the Bessel function, l is a characteristic length in the bending problem defined by

$$l = \sqrt[3]{\frac{2D_f(1-\nu_s^2)}{E_s}}. \tag{2}$$

In the situation when the plate is bonded to the half-space, Lee et al. (2009) obtained the solution to the bending problem in the form (1) provided l is modified to

$$l = \left(\frac{D_f(1+\nu_s)(3-4\nu_s)}{2E_s(1-\nu_s)}\right)^{1/3}, \tag{3}$$

but under the simplifying assumption that the in-plane stiffness of the elastic plate is sufficient to prevent any tangential displacement at the interface.

The effect of compliant substrate was studied by Lee et al. (2009) in the case of spherical indentation based on formulas (1)–(3) and using an asymptotic approximation to the integral (1) obtained by Ol'shanskii (1987). The global deformation of the plate/substrate system under a spherical indenter was approximately evaluated by means of a concentrated ring load that produces a constant deflection curvature of the plate inside the circular area.

In the present paper, we utilize the solution of the bending problem previously obtained by Bardanov and Popov (1966), which additionally takes into account the in-plane deformation of the elastic plate, and compare this solution with Eqs. (1)–(3).

However, in contrast to the approach developed by Lee et al. (2009), we approximate the global deformation as $h_1 \approx w^f(0)$, where $w^f(r)$ is the film deflection according to Bardanov and Popov (1966).

Moreover, in order to evaluate the incremental indentation stiffness

$$\frac{dP}{dh} = \frac{2aE_f}{1-\nu_f^2}k, \tag{4}$$

where k is the indentation scaling factor, which accounts for the substrate effect, we suggest to use the first-order asymptotic model

$$k_{\rm AS} \simeq \left(1 - \alpha_0 \frac{a}{t}\right)^{-1} \tag{5}$$

which was developed in (Argatov, 2010; Argatov & Sabina, 2014) under the assumption that the contact radius a is relatively small compared with the film thickness t.

The purpose of this paper is to present effective analytical formulas to account for the global deformation of the film/substrate system. In what follows, it is shown that the asymptotic constant α_0 provides the necessary information to account for the compliant substrate effect as well as in the case of relatively stiff substrate.

2. Young's modulus identification by the small-scale indentation

First of all, note that, for the sake of simplicity, formula (4) neglects the deformation of the indenter. The latter effect can be incorporated in a usual way by means of the effective elastic modulus (Johnson, 1985). Thus, in view of (5), Eq. (4) takes