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Exploring the source of non-locality in the Euler–Bernoulli and Timoshenko beam models

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ABSTRACT

In the last decade there has been significant research activity in the use of Eringen's nonlocal models to reformulate the equations of beams and plates. All of the previous works used a length scale parameter to study its effect on bending, buckling, and vibration characteristics, without identifying what the length scale parameter means. An attempt is made herein, for the first time, to relate the length scale parameter(s) to physical parameters. The Eringen's non-local Euler–Bernoulli and Timoshenko beam models are identified as continuum limits of a discrete system comprising of harmonic oscillators. The correspondence between the coefficients of the discrete and the continuum models is used to determine the source of the non-locality in the context of Eringen's non-local beams.

1. Introduction

Ever since Eringen (1972), Eringen and Edelen (1972) hypothesized that the stress at a point is not only a function of the strain at that point but also strains over the whole domain of the continuum, many researchers utilized the model to study mechanical response of a variety of continuum problems. Starting from the wave dispersion phenomena by Eringen himself (Eringen, 1983), the reformulated non-local laws have been applied to understand many important aspects in mechanics, for example elasto-plasticity, mechanical properties of carbon nanotubes, biosensors, MEMS and so on, to name a few. For various applications one may refer to Lim, Zhang, and Reddy (2015); Najar, Nayfeh, Abdel-Rahman, Choura, and El-Borgi (2010). Naturally it was important to derive the non-local versions of continuum elements like beams, plates, and shells, thanks to their usefulness in analyzing systems analytically or with drastically reduced computational overhead. In this direction, Reddy and his colleagues (Fernández-Sáez, Zaera, Loya, & Reddy, 2016; Reddy, 2007) reformulated non-local beam theories along with their solutions to study various response behaviors, including bending, vibration, and buckling. As further developments, one may refer to Reddy and El-Borgi (2014) for nonlocal beam model including moderate rotation; (Aydogdu, 2009; El-Borgi, Fernandes, & Reddy, 2015) for vibration studies of graded nano-beam and Najar, El-Borgi, Reddy, and Mrabet (2015) for the analysis of non-local beam-based eastostatic nano-actuators. Effect of non-locality in the context of non-linear analysis of Timoshenko beam model is considered in Kasirajan, Amirtham, and Reddy (2015). Nonlocal plate theory is developed in Lu, Zhang, Lee, Wang, and Reddy (2007); Reddy, Romanoff, and Loya (2016). Specifically, in Raghu, Preethi, Rajagopal, and Reddy (2016) shear deformation for nonlocal laminated plates is discussed. Nonlocal studies

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in the beams and the plates made by functionally graded materials are reported in Reddy et al. (2016) and Reddy (2014). More works in similar directions may be found in Khodabakhshi and Reddy (2015); Wang, Zhang, Ramesh, and Kitipornchai (2006).

It is by now well observed that, as the system dimension reduces, non-local macro-continuum models start varying significantly from their classical counterparts. While there has been a considerable amount of work to bring out the non-local effects in the reformulated macro-continuum models (Eringen, 1972, 1983; Eringen & Edelen, 1972; Lu et al., 2007; Ma, Gao, & Reddy, 2008; Reddy, 2007, 2010), the source of non-locality seems to be not understood well. The present manuscript attempts at filling this gap in the context of non-local Euler–Bernoulli beam theory (EBT) and non-local Timoshenko beam theory (TBT). The main idea is to identify these non-local beams as the continuum limit of a discrete system comprising of harmonic oscillators. While a phenomenon like bending is typically a manifestation of multi-layered atomic interaction, we are interested in a 1-dimensional model to capture it. Specifically, we want to find a discrete system comprised of a chain of particles that converges to beam models in the continuum limits. In this work it is identified that a system, where each such particle behaves like a harmonic oscillator, indeed approaches Eringen's non-local EBT and TBT in the continuum limits. Noting that the discrete model using harmonic oscillators successfully captures bending phenomena in the context of beam elements, similar models should be useful in describing behavior of other solid structures, which predominantly demonstrates bending behavior.

Given that the response behavior of harmonic oscillators can be described using well understood parameters, this discrete model turns out to be particularly useful in understanding the non-local parameters of the continuum models. Subsequently, we can describe a non-local parameter in the beam models as a function of well understood variables. The present work also challenges the traditional notion of incorporating a single parameter to describe non-locality in a continuum beam model by showing that there are multiple non-locality parameters and they have different origins. We observe that the non-locality parameters are not only nonlinear functions of lattice distance but they depend on other properties of the body. These facts potentially necessitate the reformulation of all the continuum models considering appropriately placed multiple non-locality parameters.

The rest of the manuscript is organised as follows. In Section 2, a discrete system is demonstrated to approach the non-local EBT in the continuum limit. In the process, the source of non-locality in the context of the EBT is identified. A similar exercise for the TBT is carried out in Section 3. Finally, some conclusions and a discussion on future extensions is presented in Section 4.

2. EBT: discrete to continuum limit

Consider a system of N harmonic oscillators, as shown in Fig. 1. The dotted lines indicate the initial positions of the particles. A force of Q_j is applied on particle p_j, resulting a displacement w_j in the direction of the force. The solid lines indicate the positions after motion ensues. We shall show shortly that the Lagrangian corresponding to the system shown in Fig. 1 describes, in the continuum limit, the response of the EBT. In this context, it is convenient to define a number of variables that will be used in writing the Lagrangian of the discrete system. The displacement in the direction of the externally applied force Q_j on the discrete particle p_j $(j = 1, 2, \ldots, N)$ is denoted by w_j, as shown in Fig. 1. Harmonic oscillator p_j vibrating with velocity v_j is assigned a mass M_0 and mass inertia I_0. Adjacent oscillators share a common mass M_2 and mass inertia I_2. The angle of rotation of the p_jth particle from its initial equilibrium position is denoted by θ_j, which characterizes the curvature angle of the particle. The corresponding radius of curvature is denoted by l_j. Each particle p_j experiences a tension $\hat{T}_j$ along the arm connecting the particle, as shown in the Fig. 2. The vertical component of $\hat{T}_j$ is denoted by T_j and the horizontal component by T_j^s. Since we assume that θ_j is small, the horizontal tension component T_j^s drops out of the analysis.

Potential energy for the discrete system of harmonic oscillators may be written as

$$\Pi_d = \sum_{j=2}^{N-1} T_j l_j \left(1 - \cos\theta_j\right) - \sum_{j=1}^{N} Q_j w_j \tag{1}$$

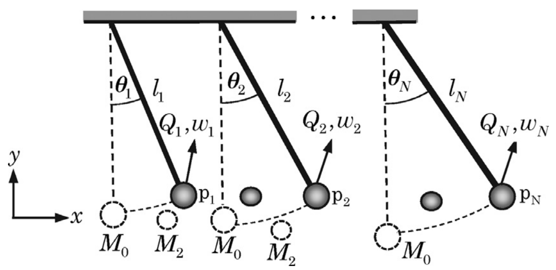

Fig. 1. A System of harmonic oscillators.