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Stress and dislocation distributions near a crack tip in ductile single crystals

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A R T I C L E I N F O

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A B S T R A C T

Within the continuum dislocation theory the asymptotic analysis of the plane strain crack problem for a single crystal having only one active slip system on each half-plane is provided. The results of this asymptotic analysis show that the square root stress singularity remains valid during the plastic deformation, while the dislocation density is proportional to the stress intensity factor and distributed inside the crystal as the square root of the distance from the crack tip. The analytical solution for the angular distribution of the dislocation density is found.

1. Introduction

Dislocations appear to reduce energy of crystals. For crystals with cracks the high stress concentration near the crack tip causes also high energy of crystals in that region. It is therefore natural to expect that, when the load is sufficiently large, dislocations nucleate near the crack tip to reduce the stress level and by this also the energy of crystals. It is then crucial to have the correct perception of how dislocations nucleate near the crack tip. Up to now, the commonly accepted point of view is that dislocations nucleate directly at the crack tip and then glide away from it under the Peach–Koehler force (Rice, 1992; Rice & Thomson, 1974). However, the analysis of crack problems reveals that the resolved shear stress is large not only at the crack tip, but also in its neighborhood. Taken this for granted, then, according to Schmid's law, dislocations must appear simultaneously in that neighborhood exhibiting the collective character of dislocation nucleation. Since the typical dislocation density is high (about $10^8 \div 10^{15}$ dislocations per square meter), it makes sense to use the continuum approach to study this problem.

This short paper aims at finding the stress and dislocation distribution near the crack tip in ductile single crystals within the continuum dislocation theory (CDT) proposed by Berdichevsky (2006a, 2006b) and developed further in Berdichevsky and Le (2007), Le and Sembiring (2008a, 2008b), Le and Sembiring (2009), Kochmann and Le (2008), Kochmann and Le (2009a, 2009b), Kaluza and Le (2011), Le and Nguyen (2012, 2013), Baitsch, Le, and Tran (2015). Considering the plane strain crack problem, we assume that during the plastic deformation only one slip system on each half-plane of the crystal is active. We provide an asymptotic analysis of this crack problem in the polar coordinates. The results of this asymptotic analysis show that the square root singularity for the stress field near the crack tip remains valid. This agrees with the singularity of

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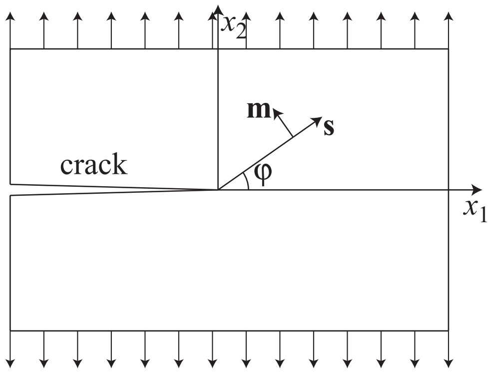

Fig. 1. The plane strain crack problem for single crystal.

HRR-field obtained by Hutchinson (1968), Rice and Rosengren (1968) in conventional plasticity for the materials with linear hardening. What the dislocation distribution inside the crystal near the crack tip is concerned, we show that they must be distributed such that the resolved shear stress is balanced with the back stress in accordance with the equilibrium of micro-forces acting on dislocations. This leads to the power law distribution $\sqrt{r}$, with r being the distance from the crack tip, for the dislocation density, with its intensity being proportional to the stress intensity factor. We find also the universal angular distribution of the dislocation density.

It is interesting to mention here the alternative approaches proposed in the existing literature. First of all, the dislocation based fracture mechanics developed by Weertman (1996) enables one to determine the dislocation distribution on the crack faces as well as inside the elasto-plastic material. However, the dislocation density found inside the crystal in this way is inconsistent in two aspects: (i) Weetman's dislocation density is defined as the gradient of the plastic strain conflicting Nye-Bilby-Kröner definition, (ii) the phenomenological elasto-plasticity is used to find the plastic strain, so the obtained dislocation distribution does not satisfy the equilibrium of micro-forces. The crack-tip fields in single crystal has been analyzed within the discrete dislocation dynamics in Van der Giessen, Deshpande, Cleveringa, and Needleman (2001). Experimental observations of the dislocation distribution near the crack tip in single crystals by electron tomography have been reported in Tanaka, Higashida, Kaneko, Hata, and Mitsuhara (2008). Another quite promising experimental method of measuring the dislocation density by using electron backscatter diffraction (EBSD) technique has been developed in Kysar and Briant (2002), Kysar, Saito, Oztop, Lee, and Huh (2010).

2. Plane strain crack problem for single crystal

Consider the plane strain problem for a single crystal containing a crack lying on the left-half of the x_1-axis as shown in Fig. 1. The depth of the crystal in the x_3-direction is taken large enough to guarantee the plane strain state having two non-zero components of displacement vector $u_1 = u_1(x_1, x_2)$ and $u_2 = u_2(x_1, x_2)$. The crystal is oriented in such a way that its lattice and mechanical properties as well as the loading condition (say tractions acting at the outer boundary) are symmetric with respect to the reflection about the x_1-axis. Because of this mirror symmetry it is sufficient to consider the upper-half of the crystal. If the load is small enough, then it is natural to assume that the crystal with this crack deforms elastically. However, if the load exceeds some threshold value, dislocations can occur causing the plastic deformation of the crystal. We assume that, during this plastic deformation, only one slip system from each half of the crystal is active and the dislocations are straight lines parallel to the x_3-axis. For the more realistic crack problems in single fcc and bcc crystals having several active slip systems the reader may consult (Rice, 1987). Letting $\mathbf{s} = (\cos\varphi, \sin\varphi, 0)$ denote the slip directions, and $\mathbf{m} = (-\sin\varphi, \cos\varphi, 0)$ the normal vector to the slip planes of the slip system in the upper-half of the crystal, we may express the plain strain plastic distortion tensor in the form $\boldsymbol{\beta} = \beta(x_1, x_2)\mathbf{s}\otimes\mathbf{m}$. We are going to determine the displacements $u_1(x_1, x_2)$, $u_2(x_1, x_2)$, and the plastic slip $\beta(x_1, x_2)$ near the crack tip during this plastic deformation.

For the plane strain state the non-zero in-plane components of the symmetric strain tensor $\boldsymbol{\varepsilon} = \frac{1}{2}(\nabla\mathbf{u} + \mathbf{u}\nabla)$ are

$$\varepsilon_{11} = u_{1,1}, \quad \varepsilon_{12} = \varepsilon_{21} = \frac{1}{2}(u_{1,2} + u_{2,1}), \quad \varepsilon_{22} = u_{2,2}.$$

Throughout the paper the comma standing before an index is used to denote the partial derivative with respect to the corresponding coordinate. The in-plane components of the symmetric plastic strain tensor $\boldsymbol{\varepsilon}^p = \frac{1}{2}(\boldsymbol{\beta} + \boldsymbol{\beta}^T)$ equal

$$\varepsilon^p_{11} = -\frac{1}{2}\beta\sin 2\varphi, \quad \varepsilon^p_{12} = \varepsilon^p_{21} = \frac{1}{2}\beta\cos 2\varphi, \quad \varepsilon^p_{22} = \frac{1}{2}\beta\sin 2\varphi.$$