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# Nonlinear vibration of axially functionally graded tapered microbeams

Navvab Shafiei*, Mohammad Kazemi, Majid Ghadiri

*Faculty of Engineering, Department of Mechanical Engineering, Imam Khomeini International University, Postal code: 34149-16818 Qazvin, Iran*

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ABSTRACT

Solving the nonlinear governing equations of a non-uniform micro- and nano-beam is a complicated challenge for researchers. For the first time, the nonlinear size-dependent vibration of a non-uniform axially functionally graded (AFG) microbeam is studied in this paper. The microbeam is modeled according to the Euler–Bernoulli beam theory and the modified couple stress theory with von-Kármán's geometric nonlinearity. The boundary conditions are considered as clamped, simply supported and clamped-simply supported. To derive the equations and boundary conditions, Hamilton's principle is utilized and then the governing equations are solved by the generalized differential quadrature method (GDQM) and direct iterative method. Finally, the effects of nonlinearity, small-scale parameter and rates of cross-section change on the fundamental and the second frequencies of the AFG, pure ceramic and pure metal microbeams are presented. It is shown that the effects of the rate of cross-section change of the microbeam along one direction depend on the nonlinearity and also the rate of cross-section change along the other direction. The results of this study can be used in designation of many microstructures such as micro electro mechanical systems (MEMS), micro-actuators, etc.

## 1. Introduction

The deviations of the experimental results from the classical continuum-based studies have proved that lacking the internal length scale parameters makes the classic theories incapable of predicting the behavior of structures in micro- and nano- scales accurately. Thus, a number of theories have been introduced by scientists to define micro-systems behavior (Koiter, 1964; Mindlin & Tiersten, 1962; Mindlin & Eshel, 1968; Toupin, 1962; Yang, Chong, Lam, & Tong, 2002) introduced a strain gradient theory which is one of the first higher order continuum models that is involved with five micro-scaled size dependent constants. A strain gradient theory of rate independent plasticity was developed by Fleck, Muller, Ashby, and Hutchinson (1994) using the dislocation theory. Moreover a strain gradient elastic bending theory was proposed for plane-strain beams by Lam, Yang, Chong, Wang, and Tong (2003). The couple stress theory was first introduced and developed by Koiter (1964), Mindlin and Tiersten (1962) and Toupin (1962) and contains four material constants. The couple stress theory which uses only conventional equilibrium relations of forces and moments of forces was employed to study materials by Cosserat and Cosserat (1909) and Voigt (1887). The couple stress theory considers the effects of a couple per unit area on a material volume along with the classical direct and shear forces per unit area. In couple stress theory, rotation is a

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* Corresponding author. Tel.: +98 935 930 8331.
*E-mail address:* navvabshafiei@gmail.com (N. Shafiei).

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variable to describe curvature while the strain gradient theory uses strain for curvature description. Recently, a modified couple stress theory is developed by Yang et al. (2002) using a symmetrical couple stress tensor.

Dynamic analysis of micro- and nano-structures has been the focus of many studies in the literatures and several investigations are performed on mircobeams (Farokhi, Ghayesh, & Amabili, 2013; Ke, Wang, Yang, & Kitipornchai, 2012; Kong, Zhou, Nie, & Wang, 2008; Mohammad-Abadi & Daneshmehr, 2014; Park & Gao, 2006; Şimşek, 2010a; Şimşek & Reddy, 2013; Tang, Ni, Wang, Luo, & Wang, 2014a, 2014b; Wang, Xu, & Ni, 2013; Xia, Wang, & Yin, 2010), non-uniform microbeams (Park & Gao, 2008; Shafiei, Kazemi, & Fatahi, 2015a, 2015b) and microbars (Akgöz & Civalek, 2014; Mustapha & Ruan, 2015).

Functionally graded (FG) materials which were first proposed in 1984 are new generation of materials with varying properties along a direction which is due to their changing volume fractions of the constituents. Studies on the FG beams are complex due to the varying mechanical behavior through the beam and researchers have tried to derive suitable theories to study FG beams. A beam element was introduced by Chakraborty, Gopalakrishnan, and Reddy (2003) to study the thermoelastic behavior of FG beam structures. A new unified approach was presented by Li (2008) for analyzing the static and dynamic behaviors of FG beams with the rotary inertia and shear deformation included. Benatta, Mechab, Tounsi, and Bedia (2008) presented high-order flexural theories for short FG symmetric beams under three-point bending.

Due to the vast applications of FG beams, a lot of researchers have studied different FG structures with many applications in recent years. Aydogdu and Taskin (2007) studied the free vibration of simply supported FG beam. Şimşek (2010b) studied the non-linear dynamic of a FG beam considering a moving harmonic load. Khalili, Jafari, and Eftekhari (2010) introduced a mixed method to study the dynamic behavior of FG Euler–Bernoulli beams subjected to moving loads. Kocaturk, Şimşek, and Akbaş (2011) performed an investigation on the non-linear static behavior of a cantilever Timoshenko FG beam under a non-follower transversal uniformly distributed load and Nateghi and Salamat-talab (2013) studied thermal and size effects on FG microbeams.

Although FG structures are examined by a lot of researchers, axially functionally graded materials and structures are studied in limited number of researches. The material constituents and the mechanical properties of an AFG beam continuously change along the axis which makes the governing equations more complex to solve. The advantage of the AFG material in micro-scaled applications makes it reasonable to devote efforts for studying the behavior of AFG structures. Recently, a number of researchers have analyzed vibration of AFG beams. Huang and Li (2010) studied the free vibration of non-uniform AFG beams. After that, Shahba and Rajasekaran (2012) studied the free vibration of Euler–Bernoulli AFG beams using the differential transform method (DTM). Using the modified strain gradient elasticity and modified couple stress theories, Akgöz and Civalek (2011) studied bending and vibration behavior of AFG tapered Euler–Bernoulli microbeams with different boundary conditions. Akgöz and Civalek (2013) studied vibration response of non-uniform AFG microbeams. Huang, Yang, and Luo (2013) presented a new method for analyzing the vibration behavior of AFG Timoshenko beams with non-uniform cross-section. Rajasekaran (2013) used the DTM to study the free bending vibration of rotating AFG Timoshenko tapered beams. Buckling of simply supported AFG beams was analyzed by Elishakoff and Guede (2004).

Recently, a number of researchers made attempts to study the nonlinear behavior of FG structures. The size effect on the nonlinear vibration of a FG microbeam was examined by Jia, Ke, Feng, Yang, and Kitipornchai (2015) under the combined electrostatic force, temperature change and Casimir force. Ebrahimi and Zia (2015) made an attempt to study the large-amplitude nonlinear vibration of FG Timoshenko beams made of porous material. Niknam, Fallah, and Aghdam (2014) studied the non-linear bending of tapered FG beam under thermal and mechanical loads. Large displacement static and transient behavior of FG curved beams was studied by Kurtaran (2015). Considering surface effect, Ansari, Pourashraf, and Gholami (2015) presented an exact solution for the nonlinear forced vibration of FG nanobeams. Most recently, Wu, Yang, and Kitipornchai (2016) studied the nonlinear vibration behavior of imperfect shear deformable FG carbon nanotube-reinforced composite (FG-CNTRC) beams based on the first-order shear deformation beam theory and von-Kármán geometric nonlinearity. Komijani, Esfahani, Reddy, Liu, and Eslami (2014) analyzed the buckling and post-buckling and vibrations in the pre/post-buckling regimes of FG beams which is resting on a nonlinear elastic foundation and subjected to in-plane thermal loads. Hemmatnezhad, Ansari, and Rahimi (2013) employed the finite element formulation and the von-Kármán nonlinear strain–displacement to study the large-amplitude free vibration of FG beams. Wattanasakulpong and Ungbhakorn (2014) performed linear and nonlinear vibration analysis of elastically end restrained FG beams. Nonlinear vibration of non-classical beam model was studied based on Eringen's nonlocal elasticity theory by Şimşek (2014). Hosseini-Hashemi, Nazemnezhad, and Bedroud (2014) used nonlocal elasticity to study the surface effects on nonlinear free vibration of FG nanobeams. Ke et al. (2012) studied the nonlinear free vibration of FG microbeams on the basis of the modified couple stress theory and von-Kármán geometric nonlinearity. In addition to the mentioned studies, several other investigations are performed on nonlinear behavior of FG beams (Fallah & Aghdam, 2011; Machado & Piovan, 2013), plates (Malekzadeh & Monajjemzadeh, 2015; Neves et al., 2011) and shells (Du & Li, 2014; Shen & Yang, 2015).

Microbeams can be considered as almost the most important elements in MEMS and have many other applications such as semiconductors, biomedical devices, etc. In real life applications, the microbeams have nonlinear vibrational behavior (Younis, 2011) which necessitates accurate nonlinear studies to have a realistic understanding for designation of microstructures which utilize microbeams. It is seen that the nonlinear size-dependent vibration of non-uniform axially functionally graded beams which have varying mechanical properties along the axis has never been studied in literature. This paper for the first time examines the nonlinear size-dependent vibration behavior of a non-uniform AFG Euler–Bernoulli microbeam with clamped, simply supported and clamped-simply supported boundary conditions based on the couple stress theory and von-Kármán's geometric nonlinearity. The Hamilton's principle is used to derive the governing equations and boundary