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Multiscale simulation of acoustic waves in homogenized heterogeneous porous media with low and high permeability contrasts

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ABSTRACT

Acoustic behavior of fluid saturated, heterogeneous, rigid porous media which consist of two micro-porous materials with high permeability contrast is studied using a two-scale model based on the homogenization theory. At the mesoscopic level, the fluid motion is governed by the generalized Darcy flow model. At the macroscopic scale, effective acoustic properties are described by a model which has recently been derived using the periodic unfolding method (Rohan, 2013).This paper presents a computational procedure for estimating the pressure and the velocity fields at both the mesoscale and the macroscale. The model is validated by comparing the multiscale solutions with those of the model with exact periodic mesostructural geometry. The numerical solutions are computed *via* the finite element method. The ranges of applicability of homogenized models with "high contrast" and "low contrast" permeabilities are also investigated.

1. Introduction

In a general setting, fluid-saturated porous media are constituted by a solid skeleton in which the fluid-saturated pores are distributed as a connected pore network, or as densely distributed particles. The porous media theory has widely been applied to describe the mechanical behaviors of various materials such as soils, rocks, wood, bones, or industrial foams. Using the linear theory of dynamics, the problem of poroelastic wave propagation in a fluid-saturated continuum has initially been developed by Biot (1956a, 1956b) by using a phenomenological approach in his two seminal papers published in 1956. The Biot-type equations that govern the flow of fluids through porous media can be shown to be approximations within the context of the theory of mixtures. A comprehensive overview of the theoretical foundations of modeling porous media using this approach can be found in the book by Rajagopal and Tao (1995) (see also the review paper by Rajagopal (2007)). The modern presentation of this theory dates back to 1957 when Truesdell (see Truesdell, 1957a, 1957b) proposed balance equations appropriate to mixtures irrespective of their constitution. Within this general framework, detailed results were elaborated in the seminal book of Truesdell and Toupin (1960) (see Sections 158, 159, 215, 243, 254, 255, 259 and 295). Ideas of this theory have then been revised by Atkin and Craine (1976). In particular, the mixture theory has been used

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by Bowen (1980, 1982) to formulate models of flow in incompressible and compressible porous media. Rajagopal and Tao (1992) have studied the propagation of waves in homogeneous isotropic and transversely isotropic elastic solids infused with fluids, within the context of this theory. More recently, the same topic was treated also in the framework of the mixture theory by Wilmanski (2006).

Homogenization procedures (see *e.g.* the book of Auriault, Boutin, and Geindreau (2010) for a mechanical point of view or Hornung (1996) for a mathematical point of view) present a complementary approach to the mixture theory. They are based on a micromechanical description of relevant interactions (*i.e.* fluid–solid interaction in our case) and typically require more information about the microstructure to be involved in the modeling. In general, it can lead to the same structure of models, as the one obtained using the phenomenological approach, but the governing equations are very complicated, if one wants to take into account non-linear constitutive representations for the interacting continua, inter-conversion of the species, complex interaction forces that need to be taken into consideration in the balance of linear momentum. In fact, many of the equations that can be arrived at by different approximations have not been rigorously derived within the context of homogenization.

Both the above mentioned treatments lead to macroscopic wave equations, whereby the feasible wavelengths are significantly larger than the microstructure sizes, *i.e.* the characteristic sizes of pore and skeleton. However, many natural/artificial porous materials exhibit the presence of heterogeneity at scales much larger than microstructure scales, but much smaller than the wavelengths. In a such porous medium with heterogeneity at the meso-scale, pore fluids in regions of dissimilar properties respond differently with changes in their fluid pressures. It significantly affects the velocity dispersion and attenuation, referred to as mesoscopic losses due to wave-induced flows (Pride, Berryman, & Harris, 2004). In principle, for simulating the wave propagation in such mesoscopic heterogeneous media, continuum porous model with spatially varying coefficients may be used. However, such a straightforward approach may be very inefficient in practice, namely in situations when the domain of interest is much larger than the heterogeneity's size. For media with periodically distributed inhomogeneities, macroscopic effective media can be derived from the continuum equations established at mesoscale. In works of Berryman and Wang (2000) and Pride and Berryman (2003a, 2003b), effective medium parameters have been derived by using volume-averaging technique for acoustic problem featured by the so-called "double porosity" media which consist of two linear isotropic porous constituents. The equations of motion have been derived by Ba, Carcione, and Nie (2011) for a double porosity medium in which spherical poroelastic inclusions are embedded in a host medium having different porosity, permeability, and compressibility.

It is important to mention that in the literature, the notion of the "double porosity" has been used in at least two different contexts, related to different classes of material structures and, thereby, giving rise different conceptions of the modeling. The first class comprises materials that consist of a fluid-filled phase interacting with a skeleton which itself is a microporous medium. This problem was first studied by Barenblatt, Zheltov, and Kochina (1960) in geomechanical context for studying flows in fractured rocks which were modeled by a system of two porous materials: the first one being associated with the fractures and the other one with the porous matrix. Thus three scales characterizing the micropores, the fractures, and the macroscopic medium were considered. As the key ingredient of the model, two average fluid pressures were defined at any spatial positions in each porous system. The acoustic waves in such double porosity media has also been studied by using the homogenization methods (see *e.g.* Auriault & Boutin, 1994; Boutin, Royer, & Auriault, 1998). In those papers, the porous material is constituted hierarchically: at the mesoscopic scale, the microporous material characterizes the matrix fractured by mesocopic fluid-saturated pores, whereby upscaling the viscous fluid flow (in rigid, or deforming skeletons) is considered with different scaling ratios influencing the asymptotic behavior of the model. This leads correspondingly to different inter-scale coupling associated with the double porosity, as highlighted by Olny and Boutin (2003). By using these models, Venegas and Umnova (2011) and Chevillotte, Perrot, and Guillon (2013) have investigated the acoustical properties of double porosity materials by using analytical and numerical methods. The obtained results were compared with experiments for some mineral foam samples.

The present work deals with another characterization of a "double porosity" medium as a periodic mixture of two different porous media which occupy two disjoint subdomains at the mesoscopic scale. In this paper, the domain Ω is a periodic mixture consisting of two disjoint rigid porous materials denoted by Ω_c and Ω_m following the notations introduced in Rohan (2013). The porous media in two subdomains Ω_c and Ω_m generate the spatial heterogeneity of permeability which will be in the focus of our study (see Fig. 1). In particular, we consider differences in the intrinsic permeability magnitudes between two components. When these permeability are not in the same order of magnitude, the material will considered as the "high permeability contrast" (HPC) porous medium; otherwise, it will be considered as the "low permeability contrast" (LPC) one.

In a recent work, Rohan (2013) has applied the periodic unfolding homogenization method to derive rigorously a two-scale model describing the acoustic wave propagation in the HPC porous medium. By introducing a scaling of the permeability, a new supplementary macroscopic quantity representing the dynamic compressibility (a counterpart to the usual effective compressibility) appears in the macroscopic wave equation. The preliminary study presented in Rohan (2013) of homogenized multilayer media has shown that homogenized models with or without the use of the permeability scaling may lead to very different macroscopic phase velocities and attenuations. However, the validation of the proposed homogenized model has not been investigated.

This paper extends the previous work of one of authors and has a twofold objective. First, we present a complete computational procedure for multi-scale analysis of acoustic wave propagation in a periodic HPC porous medium. The homogenized