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Elastic field in composite cylinders made of functionally graded coatings

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ABSTRACT

In this paper, the elastic field is studied in a composite cylinder made of heterogeneous coatings. The coating layers are such that the mechanical properties of the entire composite structure change across the radius. Thus, both Young's modulus and Poisson's ratio vary continuously in coatings across the radius. The effect of varying Young's modulus has been shown to be significant on the elastic field in these types of composites. Herein, it is shown that varying Poisson's ratio also plays an important role in the stress and strain distribution and should be considered in the design process especially in carbon coated titanium cylinders, where Poisson's ratio changes between zero and a half in the structure. In addition, variation of Poisson's ratio across the radius not only changes the elastic field, but also different spatial variations of Poisson's ratio lead to distinct hoop stresses and radial displacements for a coated cylinder. The effects of Poisson's ratio variation on the elastic field in thick composite cylinders are also studied. Two closed form expressions are derived for the normalized hoop stresses at the inner and outer surfaces of thick-walled coated cylinders. It is found that the effect of varying Poisson's ratio on the normalized hoop stress in thick-walled coated cylinders is also important and the maximum hoop stress changes dramatically for thicker cylinders.

1. Introduction

Due to excellent wear resistance, superior tribological properties, and high hardness, nanocomposite coatings have drawn scientific attentions in recent years for different applications from transport industry to energy sector (Mohammadi & Salimi, 2007; Saha & Khan, 2009; Saha, Khan, & Glenesk, 2009; Singh et al., 2010). These thin films can be deposited on metal substrates using different methods such as chemical vapor deposition (Gupta & Talha, 2015), physical vapor deposition (Sobczak & Drenchev, 2013), magnetron sputtering (Surmeneva et al., 2014), and centrifugal sintered-casting (Birman & Byrd, 2007; Kunimine, Shibuya, Sato, & Watanabe, 2015). Using these nanocomposites leads to the formation of a class of heterogeneous solids called functionally graded materials (FGMs) (Salehipour, Sahidi, & Nahvi, 2015). Since the composition and the morphology of these solids gradually change over the volume, the elastic properties of these composites are not constant and change with position (Mohammadi & Dryden, 2008). Of particular interest is a special case where the properties of these heterogeneous composites only change across the radius and not in the tangential direction (Birman, 2014).

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In most of the work dealing with the elastic field in functionally graded pipes, rings, tubes, and beams, only Young's modulus has been assumed to vary with position and Poisson's ratio has been held constant, see e.g. Tutuncu (2007), Batra (2008), Ke, Yang, Kitipornchai, and Wang (2014), Jabbari, Nejad, and Ghannad (2015) and Wang, Zhang, Xia, Wu, and Liu (2015). However, Poisson's ratio is not constant and can change with volume fraction especially in damage tolerant ceramic coatings and cellular solids (Akbarzadeh, Fu, Chen, & Qian, 2014; Wo et al., 2013). Using pulse-echo technique along with elastic impact testings, Marur and Tippur (1998) measured the variation of Young's modulus and Poisson's ratio with position in an epoxy-based glass-sphere filled particulate composite with linear graded properties. The variation of Poisson's ratio was not significant in their measurements. However, Haque and Choy (2000) reported a drastic change in Poisson's ratio for SiC monofilaments on Ti-based metal matrix composites. Poisson's ratio gradually changed almost between 0 and 0.5 in their investigations (Haque & Choy, 2000). In addition, Poisson's ratio can also change significantly with temperature variation. Evertt and Miklowitz (1944) have experimentally measured the variation of Poisson's ratio with temperature for several metals. Baltrukonis (1959) studied the thermoelastic problem of a thick-walled tube considering varying Poisson's ratio and Young's modulus in different temperatures.

Initial attempts to consider the effects of varying elastic coefficients on (thermo) elastic fields in FG problems were made by Lutz and Zimmerman (1996) and Zimmerman and Lutz (1999). Using a stochastic analysis, Yang, Liew, and Kitipornchai (2005) studied an FG plate under lateral loading and temperature. In their analysis, Poisson's ratio was not constant and its variation had some effects on the overall deflection of the FG plates (Yang et al., 2005). Mohammadi and Dryden (2009) showed that the spatial variation of Poisson's ratio significantly affects the radial displacement and stresses in FG pipes. In their contribution, Young's modulus and Poisson's ratio are related using a simple law of mixtures. They used exponential functions to identify spatial distribution of Poisson's ratio and Young's modulus. Mohammadi, Dryden, and Jiang (2011) studied stress concentration factor around a circular hole in a FG plate. Using the same method presented in Mohammadi and Dryden (2009), both Young's modulus and Poisson's ratio changed with position in their work (Mohammadi et al., 2011). Recently, Xin, Dui, Yang, and Zhang (2014) proposed a method based on the volume fraction of the constituents and studied the elastic field in a thick FG tube under internal pressure. In their contribution, both Young's modulus and Poisson's ratio were allowed to vary across the radius (Xin et al., 2014). Simsek and Reddy (2014) investigated the bending and vibration of FG microbeams using a new higher-order beam theory. In their contribution effective Young's modulus and Poisson's ratio are not constant and vary with position (Simsek & Reddy, 2014). Later on, Mohammadi (2015) studied the effects of varying Poisson's ratio upon the thermoelastic field in FG axisymmetric bodies. Significant influence of varying Poisson's ratio on the thermoelastic field was observed by the author. Akbarzadeh and Chen (2013b), Akbarzadeh and Pasini (2014) and Akbarzadeh, Abedini, and Chen (2015) considered functionally graded smart cylinders, where not only Young's modulus and Poisson's ratio but also all the other multiphysical properties vary spatially through the medium. To avoid the complicated numerical procedure, they have adopted the piecewise homogeneous layer to simulate FG smart materials.

In this paper, the coated cylinder is considered as an axisymmetric composite and the general formulation of the elastic stress and displacement fields for axisymmetric problems in polar coordinates is obtained. Both Young's modulus and Poisson's ratio are allowed to vary across the radius. Furthermore, general solutions for the governing differential equations are obtained for a coated composite cylinder. It is considered that, the cylinder is made of a special type of FG coating layers, where Poisson's ratio can vary between zero 0 and 0.5. Subsequently, the effect of varying Poisson's ratio upon the elastic field in a coated cylinder, when Poisson's ratio is defined independently, is discussed. Radial and hoop stresses along with radial displacement are calculated and the results are then expanded to thick coated cylinders. Finally, closed form expressions for maximum hoop stress both on the inner and outer surfaces of thick-walled coated cylinders with varying Poisson's ratio are obtained.

2. Basic elastic formulation

Considering the equation of motion of elastic bodies in polar coordinates (Timoshenko & Goodier, 2010), two governing differential equations in terms of stress function and radial displacement are obtained for axisymmetric bodies. In order to decrease the complexity of the formulation, the analysis is focused on radially symmetric loadings. The elastic axisymmetric solid is heterogeneous and isotropic; so that, both Young's modulus and Poisson's ratio vary only across the radius (Jabbari, Sohrabpour, & Eslami, 2002). Thus, Young's modulus and Poisson's ratio are written as

$$E = E(r), \tag{1}$$

$$\nu = \nu(r). \tag{2}$$

It is considered that, the elastic body is in plane strain condition. Thus, Young's modulus and Poisson's ratio can be tailored to represent this situation as follows:

$$E = \frac{E}{1-\nu^2}, \tag{3}$$

$$\nu = \frac{\nu}{1-\nu^2}. \tag{4}$$