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Effect of stress-softening on the ballooning motion of hyperelastic strings

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Dedicated to Millard F. Beatty, Emeritus Professor, University of Nebraska-Lincoln, Lincoln NE, USA.

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ABSTRACT

This work concerns nonlinear dynamic response of a stress-softened neo-Hookean rubber cord fixed at one boundary and undergoing constant speed circular motion at the other giving rise to ballooning motions. The steady state results are compared with that for the elastic string. The effect of material nonlinearity shows difference in these results for the single loop balloon even though the stretching deformation is not significantly high. This difference grows if the speed of rotation is increased further. Linear stability analysis shows that the single loop balloons are always stable, whereas, the stability of multi-loop balloons depends on the maximum preconditioning stretch. In usual case, one and half loop balloons are divergent unstable. Flutter instability of two loop balloons are also observed. It is shown that for certain precondition stretch, one and half loop and two loop balloons can be made completely stable. This result is in contrast to that of a linear elastic string. Thus preconditioned stretch of the rubber cord serves as a passive control parameter for stabilization of multi-loop balloons. Finally, experiments are conducted for both virgin and stress-softened rubber cords to verify the steady state and the stability results.

1. Introduction

A rope, rubber band, wire and polymeric fiber are common examples of thin, essentially lineal bodies which are described by a straight or curved spatial line. These one-dimensional material objects have negligible bending stiffness. Often the manufacturing process of such bodies or cords like polymeric fibers, yarn and natural fibers require material rotation in the packaging stage where the tension needs to be maintained at predetermined values. The rotation of the cord induces an imaginary surface generated by the body known as a balloon. The term 'balloon' is quite common in ring spinning, looming and filament winding processes.

In the book Antman (2005), the author includes general studies on large motion of flexible strings including the whirling problems. The equations of motion are derived and various interesting cases are presented. Numerous other works have been reported in the existing literature on various aspects of ring spinning of textile threads. Among these, the work of De Barr and Catling (1965), Batra, Ghosh, and Zeidman (1989a, 1989b), Fraser (1993), Stump and Fraser (1990, 1995, 1996) may be mentioned. Other similar work on either inextensible or isotropic linear elastic cords may be found in Padfield (1958), Shih (1975), Hall, Zhu, and Rahn (1995), Zhu, Sharma, and Rahn (1997), Clark, Fraser, Sharma, and Rahn

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(1998), Zhu, Hall, and Rahn (1998), Zhu and Rahn (2000). The idealized ballooning motion of a flexible Euler's elastica in a textile application problem was studied by Zhu et al. (1998). In another study (Zhu et al., 1997), they extended their inextensible model to the linear elastic model for a planar balloon in the absence of air drag. Later, various nonlinear dynamical phenomena of the complex motion considering air drag was studied by Clark et al. (1998), Zhu and Rahn (2000).

In the wet spinning process of polymer fibers the ballooning motion occurs in the final packaging stage. The polymeric fibers are stretched, washed and dried before packaging. Thus, one may conclude that the polymeric fibers are preconditioned before the final packaging operation and the material exhibits the preconditioning effect (Mullins stress-softening effect) in the ballooning motion.

When a rubber specimen is given uniaxial extension from its virgin state to a certain level of strain, unloaded and then loaded again, the stress required to produce the same stretch during reloading is smaller than that required during the primary loading phase. This phenomenon is known as the Mullins *stress softening* effect (Mullins, 1947; Mullins & Tobin, 1957) and the primary cause for such behavior is thought to be due to internal damage taking place during deformation of filled or unfilled rubberlike materials. Gurtin and Francis (1981) provided a rate dependent damage model for isotropic one dimensional hyperelastic continuum. Improved phenomenological constitutive relations are also provided by Govindjee and Simo (1991), Johnson and Beatty (1993), Ogden and Roxburgh (1999), Dorfmann, Fuller, and Ogden (2002), Zúñiga and Beatty (2002), Horgan, Ogden, and Saccomandi (2004), Qi and Boyce (2004), Zúñiga (2005), De Tommasi, Puglisi, and Saccomandi (2006). In Zúñiga and Beatty (2002), the phenomenological damage model introduced by Mullins and Tobin (1957) to characterize the uniaxial stress-softening behavior of rubberlike materials is introduced, who proposed a microstructural damage model for a filler reinforced vulcanizate described as a two phase material composed of hard phase and soft phase microstructures. The hard phase may be regarded as chain filler bonds in filled vulcanizates, or chain clusters and network entanglements; and the soft phase is the rubbery phase. This state of material is indicated as the virgin material in Zúñiga and Beatty (2002). The total amount of hard phase initially present in the microstructure is less than the amount of soft phase. During deformation of the virgin material, some of the hard phase undergoes damage and is instantly transformed to an equivalent new soft phase. The current amount of the softened part increases continuously during loading; otherwise, it remains fixed at its largest previous value determined by the maximum previous strain intensity prior to unloading. It is assumed that the unloading and reloading path is the same so long as the magnitude of the strain does not exceed its greatest previous value of strain. It turns out that Zúñiga and Beatty (2002) provides one parameter constitutive model for the stress-softened material, which is easy to use in the present dynamical problem and involves less complexity in the development of the theory of ballooning motion of a finitely deformable hyperelastic cord.

In this work, therefore, we consider the ballooning motion of a homogeneous, incompressible, isotropic and perfectly flexible stress-softened hyperelastic string characterized by Zúñiga and Beatty constitutive model (Zúñiga & Beatty, 2002). To the knowledge of the present authors, such work does not exist in literature. The geometrical boundary conditions are the same as those of Zhu et al. (1998) and we compare the numerical results with those for the linear elastic string given in Zhu et al. (1997). In particular, we investigate the effect of stress-softening on the ballooning motion of a hyperelastic rubber cord.

The dynamical problem of ballooning motion for a stress-softened neo-Hookean rubber cord is formulated in Section 2 and normalized equations of motion are obtained along with the boundary conditions. Section 3 discusses the steady state results. The stability of the steady state solutions are presented in Section 4 and the result of experimental study in Section 5. Finally, Section 6 presents the summary. We begin with the presentation of the mathematical formulation.

2. Problem formulation

Fig. 1 shows the schematic representation of a ballooning hyperelastic rubber string, which is assumed to be a uniform, perfectly flexible, homogeneous, incompressible and isotropic material body having circular cross-sectional area A, length L_0 and Young's modulus E in the undeformed state.

The undeformed homogeneous configuration is κ_0. In the absence of body force, the steady-state configuration of the cord is represented by κ_1 and its disturbed configuration is represented by κ_2. The unit vectors, $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ are oriented along the

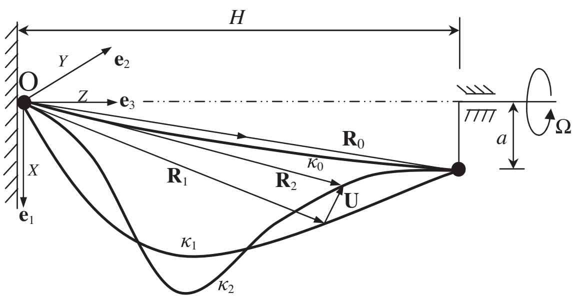

Fig. 1. Schematic diagram of a ballooning string.