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Chaotic motion of a parametrically excited microbeam

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ABSTRACT

The complex sub and supercritical global dynamics of a parametrically excited microbeam is investigated with special consideration to chaotic motion. More specifically, for a microbeam subject to a time-dependent axial load involving a constant value together with a harmonic time-variant component, the bifurcation diagrams of Poincaré sections of the system near critical point are constructed when the amplitude of the longitudinal load variations is varied as the control parameter. In terms of modelling and simulations, the small-size-dependent potential energy of the system is constructed by means of the modified couple stress theory and constitutive relations. Continuous expressions for the kinetic energy and the energy dissipation mechanism are also constructed. A transformation to a high-dimensional reduced-order model is performed via use of an assumed-mode method as well as the Galerkin scheme. A direct time-integration method is employed to solve the reduced-order model. For different cases in the sub and supercritical regimes, but close to the critical mean axial force, the bifurcation diagrams of Poincaré sections are constructed as the amplitude of the axial load variations is chosen as the bifurcation parameter. The complex dynamical behaviour of the system is analysed more precisely through plotting time traces, fast Fourier transforms (FFTs), Poincaré sections and phase-plane diagrams.

1. Introduction

Microscale continuous elements (Dehrouyeh-Semnani, 2014; Dehrouyeh-Semnani, 2015; Farokhi & Ghayesh, 2015; Ghayesh & Farokhi, 2015; Kahrobaiyan, Rahaeifard, Tajalli, & Ahmadian, 2012; Karparvarfard, Asghari, & Vatankhah, 2015; Mohammad-Abadi & Daneshmehr, 2014; Mohammadabadi, Daneshmehr, & Homayounfard, 2015; Sahmani, Ansari, Gholami, & Darvizeh, 2013; Tang, Ni, Wang, Luo, & Wang, 2014b), such as microbeams, are present in many microelectromechanical systems, such as in micro energy harvesters, microswitches, airbag accelerometers, vibration and shock sensors, biosensors, and microactuators, just to name a few. In some of these applications, microbeams are subject to *longitudinal* forces; these forces, under dynamical working conditions, vary with *time*. The unsteady axial load is of a harmonic type which is superimposed to a constant (mean) value; in this paper, the axial force is assumed to be in the form of $P_0 + P_1\cos(\omega t)$, with P_0 as the mean value (or mean axial load), P_1 as the amplitude of the force variations, and ω as the frequency of the axial load (or the excitation frequency).

The dynamical behaviour of microbeams subject to a time-dependent axial load is mainly studied in two mean axial load regimes. These regimes are known as the sub and supercritical, where the threshold is called critical mean axial

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load. When the mean axial load reaches the critical value, the stability of the system is lost by divergence, loading to buckling. Another interesting feature in the dynamical behaviour of microbeams is size effects (Akgöz & Civalek, 2014; Dai, Wang, & Wang, 2015; Şimşek & Reddy, 2013); this effect may change the qualitative as well as quantitative behaviours of the system. The modified couple stress theory is employed in this paper in order to take into account small-size effects.

Many contributions on the dynamical behaviour and stability of microbeams can be found in the literature (Baghani, 2012; Ghayesh, Amabili, & Farokhi, 2013a; Ghayesh & Farokhi, 2013; Şimşek, 2010; Şimşek & Reddy, 2013; Tang, Ni, Wang, Luo, & Wang, 2014a); the first class of the literature analysed the *free* dynamics, while the second class examined the forced statics and dynamical behaviour of microbeams subject to a *time-varying transverse* force or a *constant axial* force, mainly with the aim of obtaining frequency–response curves for the former (of the second class) and post-buckling/bending amplitude for the latter case (of the second class). This paper is the first which analyses the sub and supercritical complex global dynamics of microbeams subject to a time-dependent axial load by constructing the bifurcation diagrams of Poincaré sections, with special consideration to period-*n*, quasiperiodic, and chaotic motions.

Starting the literature review with a fundamental work by Kong, Zhou, Nie, and Wang (2008), who obtained the size-dependent linear natural frequencies of a microbeam based on the Euler–Bernoulli theory. There are other linear studies in the literature, for instance, by Wang, Xu, and Ni (2013), Asghari, Kahrobaiyan, and Ahmadian (2010), and Ma, Gao, and Reddy (2008) which examined the size-dependent dynamics of microbeams on the basis of the Timoshenko theory. Salamat-talab, Nateghi, and Torabi (2012) examined the static and dynamic behaviours of a third-order shear-deformable functionally graded microbeam. Asghari, Ahmadian, Kahrobaiyan, and Rahaeifard (2010) analysed the size-dependent motion characteristics of a functionally graded microbeam via use of the modified couple stress theory. Ramezani (2012) contributed to the field by deriving the equations of motion of a Timoshenko microbeam taking into account small-size effects and geometric nonlinearities. Ansari, Gholami, Faghih Shojaei, Mohammadi, and Sahmani (2013) obtained the buckling response of a functionally graded microbeam based on a strain gradient theory. Farokhi and Ghayesh (2015) analysed the coupled dynamics and statics of a microbeam subject to a thermal loading; the mechanical properties of the microbeam were considered temperature-dependent. Medina, Gilat, and Krylov (2012) analysed the buckling response of an electrically actuated initially curved microbeams. Ghayesh, Farokhi, and Amabili (2013b) examined the pull-in characteristics and nonlinear response of an electrically actuated microbeam taking into account small-size effects.

This paper, for the first time, examines the *complex* nonlinear dynamics of a microbeam subject to a time-dependent longitudinal load for both the sub and supercritical regimes; this is accomplished by constructing the bifurcation diagrams of Poincaré sections when the amplitude of the axial load variations is varied as the bifurcation parameter. The microbeam is modelled via the modified couple stress theory. The potential and kinetic energies are constructed and inserted into Hamilton's principle; this operation results in the continuous model of the system. A model reduction procedure is performed using an assumed-mode technique and the Galerkin procedure, which yields the reduced-order model of the system. This is solved via direct time integration through use of the variable step-size modified Rosenbrock method, yielding the motion amplitude as a function of time. The amplitude of the time-dependent component of the axial load is varied and the bifurcation diagrams of Poincaré sections are constructed; the analyses include different cases near the critical value for the constant component of the axial load, for both sub and supercritical regimes. For a selected system parameters, more detailed motion characteristics, including time traces, phase-plane portraits, fast Fourier transforms (FFTs), and Poincaré maps are displayed.

2. System energy and Hamilton's principle

The system under consideration, shown in Fig. 1, consists of an Euler–Bernoulli microbeam of length L, Young's modulus E, thickness h, cross-sectional area A, second moment of area I, mass density ρ and length-scale parameter l. A planar Cartesian coordinate system with x and z as the longitudinal and transverse axes, respectively, is considered. The displacements in the x and z directions are denoted by $u(x,t)$, and $w(x,t)$, respectively; t is time. The microbeam is subject to a time-dependent axial excitation load $P_0 + P_1 \cos(\omega t)$ at the right end; both the ends are hinged.

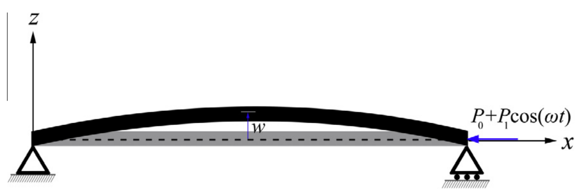

Fig. 1. Schematic representation of a microbeam subject to a time-dependent longitudinal load.