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Size dependent free vibration analysis of nanoplates made of functionally graded materials based on nonlocal elasticity theory with high order theories

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ABSTRACT

In this paper the free vibration behaviors of the nanoplate made of functionally graded materials with small scale effects are investigated. To study the small scale effects on natural frequencies, the Eringen's nonlocal theory is applied. To gain more accurate results in studying the nanoplate, higher order shear deformation plate theory (HSDT) is required when stocky and short nanoplates are considered. Employing the principle of minimum potential energy the governing equations are obtained. Generalize differential quadrature method (GDQM) is used to solve the governing equations for various boundary conditions to obtain the nonlinear natural frequencies of FG nanoplates. These models can degenerate into the classical models if the material length scale parameter is taken to be zero. Comparison between the results of GDQ method and Aghababaei and Reddy paper's for vibration of a simply supported rectangular plate made of isotropic material reveals the accuracy of GDQ method. At the end some numerical results are presented to study the effects of material length scale parameter, plate thickness, aspect ratio, Poisson's ratio boundary condition and side to thickness ratio on size dependent frequency.

1. Introduction

Nanotechnology is science, engineering, and technology conducted at the Nanoscale, which is about 1 to 100 nm. Nano science and nanotechnology are the study and application of extremely small things and can be used across all the other science fields, such as chemistry, biology, physics, materials science, and engineering. Recently, nanostructural elements such as nanobeams, nanomembranes and nanoplates have attracted worldwide attention from the researches community for their superior properties and extensive applications in nanoelectromechanical (NEM) devices. New problems at nanoscales have en-couraged the researchers to design high-performance devices such as nanosensors, nanoactuators, nanogenerators, etc. These nanoscale devices are always designed based on properties of nanobeams, nanomembranes and nanoplates (Hui & Rinaldi, 2013). At nanometer scales, size effects often become important. Both experimental and Molecular dynamics simulation results have shown that the small-scale effects in the analysis of mechanical properties of nanostructures cannot be neglected and classical continuum theories is not usable. Molecular dynamics simulation is convenient method to simulate the mechanical behavior of small size structures but it is computationally expensive for structures with large number of atoms. In the recent decades several nonclassical continuum theories that contain additional

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material length scale have been developed to overcome this barrier. Among the size dependent continuum theories, the theory of nonlocal continuum mechanics initiated by Eringen and coworkers (Eringen, 1972a, 1972b, 1983, 2002) has been widely used to analyze many problems, such as wave propagation, dislocation, and crack singularities and, from the pioneer work of Peddieson, Buchanan, and McNitt (2003), for problems involving nanostructures. Thus, the nonlocal theory of elasticity has been used to address the behavior of beams (Aranda-Ruiz, Loya, & Fernández-Sáez, 2012; Loya, Lopez-Puente, Zaera, & Fernandez-Saez, 2009; Lu, 2007a; Lu, Zhang, Lee, Wang, & Reddy, 2007b; Reddy & El-Borgi, 2014), rods (Murmu & Adhikari, 2010; Sun & Zhang, 2003), plates (Murmu & Pradhan, 2009), as well as carbon nanotubes (CNTS) (Narendar & Gopalakrishnan, 2011; Zhou & Li, 2001). Free vibration analysis of nanoplate has been intensively investigated in the literature. Recently, Reddy (2007) applied a version of nonlocal elasticity for formulating a nonlocal version of different beam theories including those of Euler–Bernoulli, Timoshenko, Levinson and Reddy beam theory to analyze bending, buckling and vibration of nanobeams. In his study, different displacement functions are chosen in the first step and then all steps are repeated when deriving beam equations of motion. Also in his study length scale effect cannot be observed (because of considering the constant beam length). Lu et al. (2007b) proposed a nonlocal plate model based on Eringen's theory and derived the basic equations for the classical and the first order plate (FSDT) theories. Aghababaei and Reddy (2009) studied the vibration of isotropic rectangular nanoplate using the nonlocal elasticity theory and the third order shear deformation theory. Pradhan and Murmu (2009) presented nonlocal elasticity theories to study the stability characteristics of single layer graphene sheets (SLGS) based on classical plate theory (CLPT) and used Levy's approach to solve the governing equations for various boundary conditions of the graphene sheets. Pradhan and Phadikar (2009) used the first order shear deformation plate theory (FSDT) and the nonlocal elasticity theory to analyze the vibration of a simply supported rectangular nanoplate. Pradhan and Phadikar (2009) presented the vibration of embedded multilayered rectangular graphene sheets considering the small scale effects. Ansari, Rajabiehfard, and Arash (2010) developed a nonlocal finite element plate model to study the vibrational characteristics of multilayered graphene sheets with different boundary conditions embedded in an elastic medium. Aksencer and Aydogdu (2011) derived governing equations of motion of FSDT plates using the nonlocal differential relations of Eringen. Navier type solution was used for simply supported plates and Levy type method was used for plates with two opposite edges simply supported and the remaining ones arbitrary. Babaei and Shahidi (2011) studied the elastic buckling behavior of quadrilateral single layer graphene sheets under biaxial compression employing nonlocal continuum and used the Galerkin method to solve the obtained equations. Narendar (2011) investigated the buckling analysis of isotropic nanoplates using two variable refined plate theory and nonlocal small scale effects and proposed the closed form solution for buckling load of a simply supported rectangular nanoplate. Hashemi and Samaei (2011) proposed an analytical solution for the buckling analysis of rectangular nanoplates and in order to extract characteristic equations of the micro/-nanoscale plate under in plane load, the analysis procedure was based on the nonlocal Mindlin plate theory.

Functionally graded materials (FGMs) are heterogeneous composite materials whose properties change smoothly and continuously along desired dimension(s). This continuously varying composition eliminates interface problems, and thus, the stress distributions are smooth (Fatehi & Nejad, 2015; Zenkour, 2013). A number of papers considering various aspects of FGM have been published in recent years (Kahrobaiyan, Rahaeifard, Tajalli, & Ahmadian, 2012; Mohammad-Abadi & Daneshmehr, 2014a, 2014b; Nejad, Rastgoo, & Hadi, 2014; Simsek & Reddy, 2013; Xue & Pan, 2013). With the advance of technology, FGMs are started to be used in micro/nanoelectromechanical systems (MEMS/NEMS),such in the form of shape memory alloy thin films with a global thickness in micro- or nano-scale (Lü, Lim, & Chen, 2009), electrically actuated MEMS devices (Zhang & Fu, 2012), and atomic force microscopes (AFMs) (Kahrobaiyan, Asghari, Rahaeifard, & Ahmadian, 2010). Natarajan, Chakraborty, Thangavel, Bordas, and Rabczuk (2012) presented application of nonlocal elasticity for 2D nanostructures, it can be mentioned to the size dependent linear free flexural vibration analysis of FG nanoplates using finite element method. Daneshmehr, Rajabpoor, and pourdavood (2014) demonstrated size dependent buckling analysis of FG nanoplates based on nonlocal elasticity theory. In this paper, higher order shear deformation plate theory (HSDT) is chosen and generalize differential quadrature method (GDQM) is used to solve the governing equations for various boundary conditions.

The aim of the present study is to investigate size effects on free vibration analysis of rectangular FG nanoplates. The Eringen's nonlocal theory is apply to study the small scale effects. The higher order shear deformation theory of Reddy (1984) is based on a displacement field that includes the cubic term in the thickness coordinate, hence the transverse shear strain and hence stress are represented as quadratic through the plate thickness and vanish on the bounding planes of the plate. Consequently, the shear correction factor is avoided in this theory. The higher order shear deformation theory yields results that are close to 3-D elasticity solutions (Reddy & Wang, 1998; Reddy, 2006). Therefore, Higher order shear deformation plate theory (HSDT) is employ to gain more accurate results in studying the nanoplate. At the end size effect, nonlocal parameter and power index on natural frequency ratios are demonstrate with numerical results.

2. Analysis

The FG thin nanoplate with length L_1, width L_2, and thickness $h$is assumed to be composed of two different phases (metal and ceramic phases) and the volume fraction of material phases are assumed to vary continuously in thickness direction, Cartesian coordinates (x, y, z) are considered (Fig. 1).

The volume fractions of the ceramic V_c and metal V_m corresponding to the power law are expressed as