



# Subcritical parametric dynamics of microbeams



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## ABSTRACT

The aim of this paper is to analyse the size-dependent nonlinear parametric dynamics of microbeams; the source of parametric excitation is a time-dependent longitudinal excitation load. Taking into account small-size effects, via the modified couple stress theory, the expressions for the potential and kinetic energies of the system are developed. A multi-degree-of-freedom discretised system is obtained by transforming the continuous model into a reduced-order one via the Galerkin scheme. For the system in the subcritical mean axial load regime, the parametric response is obtained via two different numerical techniques; first one is based on a continuation technique and the second one is via a direct time-integration method. A stability analysis is also conducted via the Floquet theory. Results for the nonlinear parametric response are illustrated in the form of parametric frequency–response diagrams, parametric force–response curves, time histories, phase-plane portraits, fast Fourier transforms (FFTs), and Poincaré maps. The effect of taking into account the length-scale parameter on the parametric response of the system is also highlighted.

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## 1. Introduction

Microbeams, as the most widely used microscale continuous elements, can be found in various microelectromechanical devices (Ghayesh, Farokhi, & Amabili, 2013; Ouakad, 2013; Wang & Soper, 2006), such as in electrically actuated microactuators, microswitches, biosensors, vibration shock sensors, and biomechanical organs. The strange size-dependent deformation behaviour of microscale elements, predicted experimentally (Fleck, Muller, Ashby, & Hutchinson, 1994; Lam, Yang, Chong, Wang, & Tong, 2003; McFarland & Colton, 2005), cannot be predicted theoretically by means of the classical continuum theory. Hence, it is essential to employ a higher-order continuum model, such as the modified couple stress (Baghani, 2012; Ghayesh, Farokhi, & Amabili, 2013; Tang, Ni, Wang, Luo, & Wang, 2014; Şimşek, 2010; Şimşek & Reddy, 2013) and strain gradient elasticity (Akgöz & Civalek, 2011; Akgöz & Civalek, 2013; Dehrouyeh-Semnani, 2014; Ghayesh, Amabili, & Farokhi, 2013; Kahrobaiyan, Rahaeifard, Tajalli, & Ahmadian, 2012; Kong, Zhou, Nie, & Wang, 2008; Akgöz & Civalek, 2013) theories, which are capable of capturing size effects and predicting this phenomenon theoretically.

In general, microscale elements, in different microdevices, are subject to *longitudinal* excitation loads, which are usually *time-dependent* under *dynamical* operating conditions. The resultant axial load is unsteady, which can be modelled via superimposing time-dependent variations on a constant value, as done in this paper. The time-dependent axial load causes the system to be classified as a *parametrically excited* system (Ghayesh, 2010, 2012a, 2012b) with notable characteristics of

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the occurrence of the *principle parametric resonance* when the frequency of the axial load variations approaches *twice* any natural frequency of the linear system. Another interesting feature is that at sufficiently large values of the mean axial load, the system loses stability via divergence; hence, the dynamics of this class of systems is primarily investigated in two main regimes of the mean value of the axial load, i.e. the *subcritical* and *supercritical*. At the threshold *critical* mean axial load, the stability is lost by divergence.

### 1.1. Literature review

The static and dynamic behaviours of microbeams have received considerable attention in recent years (Belardinelli, Brocchini, Demeio, & Lenci, 2013; Ghayesh, Farokhi, & Amabili, 2014; Ke, Wang, Yang, & Kitipornchai, 2012; Wang, Xu, & Ni, 2013). For example, Kong et al. (2008) developed a linear theoretical model for an Euler–Bernoulli microbeam, employing the modified couple stress theory, in order to obtain the natural frequencies as well as analysing the effect of the length-scale parameter. Ma, Gao, and Reddy (2008) applied the same theory to investigate the free dynamical behaviour of a Timoshenko microbeam. Asghari, Kahrobaian, Rahaeifard, and Ahmadian (2011) contributed to the field by investigating the size-dependent dynamic behaviour of Timoshenko and functionally graded microbeams on the basis of the modified couple stress theory. Ke and Wang (2011) examined the dynamic stability of microbeams made of functionally graded materials (FGMs) considering the same theory. The investigations were continued by Ansari, Gholami, and Sahmani (2011), who employed the strain gradient elasticity theory to study the dynamics of a functionally graded Timoshenko microbeam. Wang, Zhao, and Zhou (2010) employed the same theory to examine the free dynamics of a microscale Timoshenko beam. Tavallaeinejad, Eghesad, Mahzoon, Khalghollah, and Yazdi (2014) contributed to the field by developing a nonlinear rotating microbeam model based on a strain gradient elasticity theory; they discretised the equation of motion via Rayleigh–Ritz method and solved the resultant equations via a direct time-integration technique. The static buckling of an axially loaded microbeam was investigated by Akgöz and Civalek (2011, 2012), who employed both the strain gradient and modified couple stress theories. These investigations were continued by Ghayesh & Farokhi (2015b) and Ghayesh, Farokhi, and Amabili (2013), who examined the nonlinear dynamical behaviour of Euler–Bernoulli and Timoshenko microbeams, employing the modified couple stress theory.

### 1.2. Contributions of the current study to the field

All of the studies reviewed in Section 1.1 examined either the *free* dynamics of microbeams or *forced* statics and dynamics of microbeams subject to either *time-dependent transverse* loads or *constant* (i.e. time-independent) axial loads; the current paper examines the size-dependent dynamics of a microbeam under a *time-dependent axial load*. In particular, the size-dependent subcritical parametric response of a microbeam subject to a time-dependent axial load is analysed. It should be noted that the linear theory is not capable of predicting the response of the system after the occurrence of a bifurcation due to either static or dynamic axial load. Hence, in order to be able to examine the parametric response of the system and to obtain non-trivial bifurcated solutions due to time-dependent axial load, it is necessary to consider a geometrically nonlinear model, taking into account the mid-plane stretching (Abou-Ryan, Nayfeh, Mook, & Nayfeh, 1993; Emam & Nayfeh, 2009). The expressions for the kinetic and size-dependent potential energies of the system are developed using constitutive relations via employing the modified couple stress theory. A high-dimensional model reduction is performed via the Galerkin scheme, leading to a high-dimensional reduced-order model. This model is solved using an eigenvalue analysis (for the linear natural frequencies), and the pseudo-arclength continuation method as well as direct time-integration (for the nonlinear analysis). The stability of the system is analysed via use of the Floquet theory. For the system in the subcritical regime, the parametric response is obtained and presented through use of parametric frequency–responses, parametric force–responses, time histories, phase-plane diagrams, fast Fourier transforms (FFTs), and Poincaré maps.

## 2. Continuous model based on modified couple stress theory

Shown in Fig. 1 is the schematic representation of the system; i.e. a microbeam of length  $L$ , thickness  $h$ , axial stiffness  $EA$ , and flexural stiffness  $EI$ , which is subject to a time-dependent longitudinal excitation load in the form of  $P_0 + P_1 \cos(\omega t)$ .  $w(x, t)$  and  $u(x, t)$  represent the transverse and longitudinal displacements, respectively, where  $x$  is the axial coordinate and  $t$  is time;  $z$  denotes the transverse coordinate.

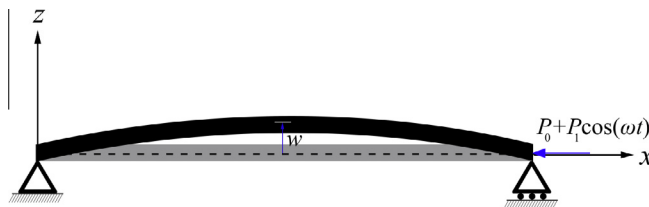


Fig. 1. Schematic representation of a microbeam subject to a time-dependent axial load.

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