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A simulation method for permeability of porous media based on multiple fractal model

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ABSTRACT

Fluid flow in fractal porous media is a ubiquitous natural phenomenon which has received much attention over three decades in a wide variety of fields. In order to find a relationship between the pore of distribution and the permeability of porous media, a simulation method for the permeability of porous media is proposed based on the multiple fractal model. The pore size distributions and the permeabilities of the porous medium simulated by the presented method are compared with available experimental data. The validity of simulation method is obtained by the good agreement between the simulated results and the experimental data.

1. Introduction

Since Mandelbrot's pioneer work (Mandelbrot, 1983), flow phenomena in fractal porous media have been studied extensively for more than three decades. Katz and Thompson (1985) proved the pore spaces are fractal geometries in several sandstones by using scanning electron microscopy and optical data. Jacquin and Adler (1987) measured some geological structures and found them were fractal. The consequences of these measurements can be used on the analysis of transport processes in porous media. Sahimi (1993) considered that fractal and percolation concepts play important roles in the characterization of porous medium, from the smallest length scale at the pore level to the largest length scales at the fracture and fault scales. Li and Logan (2001) demonstrated that cluster-fractal model was more practical to predict the permeability of the porous media than single-particle-fractal model. The cluster-fractal model has been used widely in many fields. Karacan and Halleck (2003) presented a fractal model for predicting permeability around perforation tunnels. The model provided an easier and cheaper way of mapping the permeability distribution around the perforation tunnels compared to the viscous fluid injection and pressure transient measurement techniques. Cai, Yu, Zou and Luo (2010) developed an analytical model for characterizing spontaneous imbibitions of wetting liquid vertically into gas-saturated porous media, based on the fractal character of pores in porous media. Shou, Ye, and Fan (2014) designed a single-layer porous structure to achieve the fastest capillary flow under gravity. The theoretical results obtained can be used for the optimization of porous architectures, achieving excellent liquid management properties. Xiao, Fan, and Ding (2014) obtained an analytical model for effective

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thermal conductivity of nanofluids, considering the effect of Brownian motion of nanoparticles. The researches of flow in fractal porous media have never been stopped from Mandelbrot's work (Perfect, 2005).

It is convenient to describe the characters of porous media and explain the relationship among the parameters of porous media using the fractal theory (Jiang, Wang, Hou, & He, 2013). Hansen and Skjeltorp (1988) thought most porous medium were very complex systems consisted of a range of irregular grain sizes, and that made it be hard to find the relationship between permeability and porosity. They considered that the fractal theory was helpful for the research and proposed a relationship equation between permeability and porosity using the fractal theory. Pape, Clauser, and Iffland (1999) presented a relation linking porosity to permeability based on a fractal model for the internal structure of the porous media. Good agreement between the predictions of the relation and the experimental data was obtained. Park, Lee, and Lee (2006) developed a model based on fractal theory and incorporated a cake collapse effect to predict the porosity and permeability of the cake layers. Costa (2006) derived a permeability–porosity equation using the classical Kozeny–Carman approach and a fractal pore-space geometry assumption. The equation was simple and capable to describe permeabilities of different non-granular porous media. Most studies of permeability of fractal porous media expounded the relationship between permeability and porosity (Cai, Perfect, Cheng, & Hu, 2014; Cihan, Perfect, & Tyner, 2007; Tan, Li, Zhang, Liu, & Cai, 2015).

Researchers found that the natural porous media, such as rock, soil and biological tissue, is a very complex structure, and the multifractal theory is a practical way to describe the natural porous media (Stanley & Meakin, 1988). Saucier (1992) calculated the scaling exponents of the effective absolute permeability in multifractal porous media and discuss the implications of the results on the understanding of fluid flow in oil reservoirs. Rigby and Gladden (1996) proposed a multifractal description of porous media, and represented the macroscopic heterogeneity associated with the pore-size distribution and the fractal characteristics of the microscopic pore structure. Martín and Montero (2002) analyzed the characterization of dry volume-size distributions in soils using the application of laser diffraction and multifractal theory. The result of analysis showed that multifractal is a useful mathematical tool for the characterization of porous media. Hunt and Gee (2002) proposed a "dual" fractal model using two types of fractal porous medium. The pore size distributions of some porous media were described by the "dual" fractal model. When the pore size distribution of porous media is complex, it is tough to be described accurately by two types of fractal porous medium. Perfect, Gentry, Sukop, and Lawson (2006) derived an approximate analytical expression for the effective hydraulic conductivity of multifractal porous media based on the generation of Sierpinski carpet. They found geometrical multifractals were good to simulate distinct facies or transport abilities of porous media. Vázquez et al. (2008) analyzed the pore size distributions of soils affected by rainfall based on multifractal theory. Pore size distributions of aggregates from both the reference soil surface and the soil surface disturbed by rain exhibited multifractal behavior. Paz-Ferreiro and Vázquez (2014) characterized pore size distributions from tropical soils using the multifractal approach and found that multifractal analyses of soil pore size distributions are useful for analyzing the soil porous system.

In this paper, we attempt to establish a method to describe the pore size distribution and the permeability of porous media at the same time assuming that the porous media are consisted by some types of fractal porous media. To this end, multiple fractal model will be derived in Section 2, and then a simulation method for the pore size distribution and the permeability of porous media is proposed in Section 3, finally in Section 4 relevant results and discussions are demonstrated and compared. Conclusions are given in Section 5.

2. The multiple fractal model

The natural porous media are composed of different types of porosities. For example, the main porosity types of sedimentation rocks are now described and illustrated in Fig. 1. Six main types of pore systems have been show in Fig. 1, including primary intergranular porosity, primary intragranular porosity, secondary intergranular porosity, secondary intragranular porosity, vuggy porosity and fracture porosity. So it is reasonable to assume that the porous media is composed of some types of porous medium who exhibit fractal behavior in the range of the minimum to the maximum pore size (Fig. 2). Each type of the fractal porous medium has independent fractal parameters, such as the maximum pore diameter, $\lambda_{\max i}$, the minimum pore diameter, $\lambda_{\min i}$ and the pore fractal dimension, D_{fi}, of the ith porous media.

The cumulative pore numbers, N_i, of each porous medium whose diameters are greater than or equal to the pore diameter λ follow the fractal scaling law (Mandelbrot, 1983).

$$N_i(l \geqslant \lambda) = \left(\frac{\lambda_{\max i}}{\lambda}\right)^{D_{fi}} \tag{1}$$

where $0 < D_{fi} < 2$ in the two-dimensional space and $0 < D_{fi} < 3$ in the three-dimensional Euclidian space.

Differentiating Eq. (1) with respect to λ the pore number of the ith porous media lying between λ to $\lambda + d\lambda$ on cross-sectional area can be obtained.

$$-dN_i = D_{fi}\lambda_{\max i}^{D_{fi}}\lambda^{-(D_{fi}+1)}d\lambda \tag{2}$$

It should be noted that the negative sign implies that pore number of the ith porous media decreases with the increase in pore diameter.