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Electro-thermoelasticity theory with memory-dependent derivative heat transfer

Magdy A. Ezzat[a,*], Ahmed S. El Karamany[b], A.A. El-Bary[c]

[a] Department of Mathematics, Faculty of Education, Alexandria University, Egypt
[b] Department of Mathematical and Physical Sciences, Nizwa University, Nizwa 611, P.O. Box 1357, Oman
[c] Arab Academy for Science and Technology, P.O. Box 1029 Alexandria, Egypt

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ABSTRACT

A mathematical model of electro-thermoelasticity has been constructed in the context of a new consideration of heat conduction with memory-dependent derivative. The governing coupled equations with time-delay and kernel function, which can be chosen freely according to the necessity of applications, are applied to several concrete problems: (i) a problem of time-dependent thermal shock; (ii) a problem for a half-space subjected to ramp-type heating and (iii) a problem of a layer media. Laplace transform techniques are used. According to the numerical results and its graphs, conclusion about the new theory has been constructed. The predictions of the theory are discussed and compared with dynamic classical coupled theory. The result provides a motivation to investigate conducting thermoelectric materials as a new class of applicable materials.

1. Introduction

Mathematical modeling is the process of constructing mathematical objects whose behaviors or properties correspond in some way to a particular real-world system. The term real-world system could refer to a physical system, a financial system, a social system, an ecological system, or essentially any other system whose behaviors can be observed. In this description, a mathematical object could be a system of equations, a stochastic process, a geometric or algebraic structure, an algorithm or any other mathematical apparatus like a fractional derivative, integral or fractional system of equations. The fractional calculus and the fractional differential equations are served as mathematical objects describing many real-world systems.

Thermoelectric is an old field. In 1823, Thomas Seebeck discovered that a voltage drop appears across a sample that has a temperature gradient. This phenomenon provided the basis for thermocouples used for measuring temperature and for thermoelectric power generators.

A direct conversion between electricity and heat by using thermoelectric materials has attracted much attention because of their potential applications in Peltier coolers and thermoelectric power generators (see Rowe, 1995). The interaction between the thermal and magnetohydrodynamic fields is a mutual one, owing to alterations in the thermal convection and to the Peltier and Thomson effects ($\Pi = ST$) as in Morelli (1997), where Π is a Peltier coefficient, S is thermoelectric power and T is the absolute temperature (although these are usually small). Thermoelectric devices have many attractive features compared with the conventional fluid-based refrigerators and power generation technologies, such as long life, no moving part, no noise, easy maintenance and high reliability. However, their use has been limited by the relatively low performance of present thermoelectric

* Corresponding author. Tel.: +20 1005446625; fax: +20 35835202.
E-mail address: maezzat2000@yahoo.com (M.A. Ezzat).

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Nomenclature

λ, μ	Lame's constants
ρ	density
t	time
C_E	specific heat at constant strain
κ	thermal conductivity
T	temperature
T_o	reference temperature
μ_o	magnetic permeability
ε_o	electric permittivity
σ_o	electric conductivity
σ_{ij}	components of stress tensor
u_i	components of displacement vector
C_o	$=[(\lambda+2\mu)/\rho]^{1/2}$, speed of propagation of isothermal elastic waves
η_o	$=\rho C_E/K$
θ	$=T-T_0$, such that $\lvert\theta/T_0\rvert \ll 1$
q_i	components of heat flux vector
B_i	components of magnetic field strength
E_i	components of electric field vector
J_i	components electric density vector
H_i	magnetic field intensity
S, k_o	Seebeck coefficient
Π, π_o	Peltier coefficient
e	dilatation
δ_o	non-dimensional constant for adjusting the reference
Q	the intensity of applied heat source per unit mass
α_T	coefficient of linear thermal expansion
M	magnetic field parameter
ε	thermoelastic coupling parameter
γ	$=(3\lambda+2\mu)\alpha_T$
δ_{ij}	Kronecker delta function

materials as shown in the work of Hicks and Dresselhaus (1993). The performance of thermoelectric devices depends heavily on the material intrinsic property; Z, known as the figure of merit and defined by Hiroshige, Makoto, and Toshima (2007), $ZT = \frac{\sigma_o S^2}{\kappa} T$ where σ_o, κ and S are respectively the electrical conductivity, thermal conductivity and thermoelectric power or Seebeck coefficient. Increasing of such parameter Z has a positive effect on the efficiency of thermoelectric device. In order to achieve a high figure of merit, one requires a high thermopower S, a high electrical conductivity σ_o, and a low thermal conductivity κ. However, this process is not easy as the written sentence. The direct proportion between σ_o and κ and the inverse proportion between S and σ_o yields a difficulty in improving the thermoelectric efficiency.

The theory of generalized thermoelasticity has drawn attention of researchers due to its applications in various diverse fields such as earthquake engineering, nuclear reactor's design, high energy particle accelerators, etc. Actually, as is well known, the term generalized usually refers to thermodynamic theories based on hyperbolic (wave-type) heat equations, so that a finite speed for propagation of thermal signals is admitted. Because of the experimental evidences in the support of finiteness of the heat propagation speed, Ackerman et al. (1966), the generalized thermoelasticity theories are considered to be more realistic than the conventional theory in dealing with the practical problems involving very large heat fluxes at short intervals, like those occurring in the laser units and energy channels. In this connection, several kinds of generalizations have been performed, typically modifying the entropy production inequality and/or the set of dependent and independent constitutive variables.

In the literature concerning thermal effects in continuum mechanics there are developed several parabolic and hyperbolic theories for describing the heat conduction. The hyperbolic theories are also called theories of second sound and there the flow of heat is modeled with finite propagation speed, in contrast to the classical model based on the Fourier's law leading to infinite propagation speed of heat signals. A review of these theories is presented in the articles by Sherief and Dhaliwal (1980), Chandrasekharaiah (1998), Hetnarski and Ignaczak (2000) and Ezzat and El-Karamany (2002a,b).

In the last decade, considerable interest in fractional calculus has been stimulated by the applications in different areas of physics and engineering. Recently, some efforts have been done to modify the classical Fourier law of heat conduction by using the fractional calculus in Povstenko (2005), Sherief, El-Said, and Abd El-Latief (2010) and El-Karamany and Ezzat (2011). One can refer to Podlubny (1999) for a survey of applications of fractional calculus.