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Size-dependent parametric dynamics of imperfect microbeams

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ABSTRACT

The nonlinear parametric dynamics of a geometrically imperfect microbeam subject to a time-dependent axial load is investigated in this paper. Based on the Euler–Bernoulli beam theory and the modified couple stress theory, continuous models for kinetic and potential energies are developed and balanced via use of Hamilton's principle. A model reduction procedure is carried out by applying the Galerkin scheme coupled with an assumed-mode technique, yielding a high-dimensional second-order reduced-order model. A linear analysis is performed upon the linear part of the reduced-order model in order to obtain the linear size-dependent natural frequencies. A nonlinear analysis is performed on the reduced-order model using the pseudo-arclength continuation method and a direct time-integration technique, yielding generalised coordinates, and hence the system parametric response. It is shown that, the steady-state frequency-response curves possess a trivial solution, both stable and unstable, throughout the solution space, separated by period-doubling bifurcation points, from which non-trivial solution branches bifurcate.

1. Introduction

The nonlinear dynamics of micro structures has been a matter of investigation among scientists and engineers in the last few years. Microscale continuous elements play a fundamental role in a wide variety of engineering devices and modern machine elements such as in microactuators, biosensors, microswitches, electrostatically excited micro actuators, and vibration shock sensors (Asghari, Kahrobaiyan, & Ahmadian, 2010; Baghani, 2012). The strange behaviour of microbeams was first detected through experimentations (Fleck, Muller, Ashby, & Hutchinson, 1994; Lam, Yang, Chong, Wang, & Tong, 2003; McFarland & Colton, 2005); this behaviour is highly dependent on *size*––the classical continuum mechanics is not capable of capturing size effects. However, employment of new higher-order continuum mechanics theories, such as the strain gradient (Akgöz & Civalek, 2013; Akgöz & Civalek, 2011; Dehrouyeh-Semnani, 2014; Ghayesh, Amabili, & Farokhi, 2013c; Kahrobaiyan, Rahaeifard, Tajalli, & Ahmadian, 2012; Karparvarfard, Asghari, & Vatankhah, 2015; Kong, Zhou, Nie, & Wang, 2008) and modified couple stress (Dai, Wang, & Wang, 2015; Farokhi, Ghayesh, & Amabili, 2013; Ghayesh & Farokhi, 2013; Şimşek, 2010; Şimşek & Reddy, 2013; Tang, Ni, Wang, Luo, & Wang, 2014), has led to successful predictions of the nonlinear behaviour of microscale continuous elements.

It is well-known that microscale continuous elements, such as microbeams (Belardinelli, Brocchini, Demeio, & Lenci, 2013; Farokhi & Ghayesh, 2015b; Ghayesh & Farokhi, 2015b; Ghayesh, Amabili, & Farokhi, 2013a; Ke, Wang, Yang, & Kitipornchai, 2012; Mohammad-Abadi & Daneshmehr, 2014; Wang, Xu, & Ni, 2013) and microplates (Farokhi & Ghayesh, 2015a; Ghayesh & Farokhi, 2015a) are subject to axial loads. Constant axial loads may cause buckling; however, under dynamical operating conditions,

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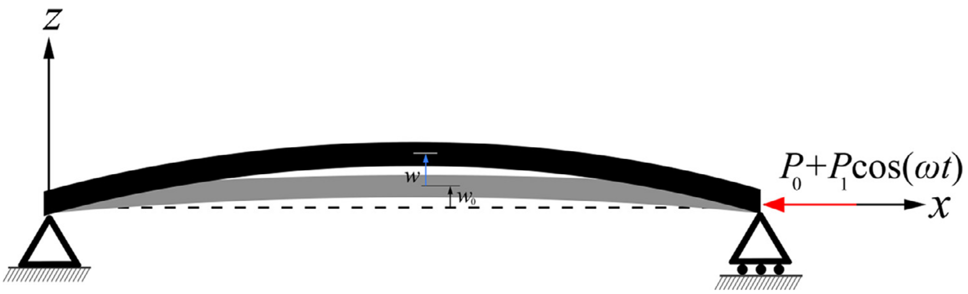

Fig. 1. Schematic representation of an initially curved microbeam subject to a time-dependent axial load.

time-dependent axial loads are generated giving rise to parametric resonances and bifurcations from trivial response; this class of systems are called *parametrically excited* (Ghayesh, 2010). Class features include the occurrence of the principal parametric resonance around *twice* the first natural frequency of the linear system.

It is worthwhile noting that *geometrically imperfect* (i.e. initially curved) microbeams are very likely to be produced due to non-ideal manufacturing process. Furthermore, in many applications, such as in microswitches, deformable electrodes are manufactured initially curved intentionally to display a snap-throng motion for on/off states. Due to these facts, considering an initial geometric imperfection in the dynamical analysis of microscale elements leads to more reliable results, as done in this paper.

There are many studies in the literature which analysed the linear and nonlinear statics and dynamics of microbeams; the systems considered were either subject to a *constant axial load* (leading to buckling) or a *time-varying transverse excitation load*. The dynamics of an imperfect microbeam under a *time-dependent axial load* has not been investigated yet in the literature; the current paper is the first to do so. A research by Kong et al. (2008) is one of the fundamental studies which obtained the natural frequencies of an Euler–Bernoulli microbeam for various length-scale parameters. Other linear investigations have been conducted, for instance, by Ma, Gao, and Reddy (2008) in order to examine the size-dependent dynamical behaviour of a Timoshenko microbeam. Roque, Fidalgo, Ferreira, and Reddy (2013) examined the size-dependent static deflection of a composite laminated Timoshenko microbeam based on the modified couple stress theory. Salamat-talab, Nateghi, and Torabi (2012) contributed to the field by analysing the statics and dynamics of third-order shear-deformable functionally graded microbeams. A three-dimensional oscillation analysis of a cylindrical microbeam was performed by Wang et al. (2013). Ramezani (2012) derived the equations of motion of a size-dependent nonlinear Timoshenko microbeam based on the strain gradient theory. Ansari, Gholami, Faghih Shojaei, Mohammadi, and Sahmani (2013) analysed the buckling behaviour of a functionally graded microbeam based on a strain gradient theory. A nonlinear model was employed by Xia, Wang, and Yin (2010) in order to develop a non-classical model for the post-buckling analysis of a microbeam. Ghayesh, Farokhi, and Amabili (2013c) analysed the dynamical behaviour of electrically actuated microbeams, with special consideration to size-dependent pull-in instabilities.

To the authors' best knowledge, there is no study in the literature which studied the *nonlinear size-dependent dynamics of a geometrically imperfect microbeam subject to a time-dependent axial load* on the basis of the modified couple stress theory. As opposed to transversely excited (perfect) systems, this class shows a nonlinear parametric response in the vicinity of the twice any linear natural frequency with period-doubling bifurcations. More specifically, based on the modified couple stress theory, the size-dependent potential energy of the system is obtained; the kinetic energy is also developed as a function of the displacement field. These expressions are inserted into Hamilton's principle leading to the continuous model of the system. The Galerkin scheme is applied to the continuous model and a high-dimensional reduced-order model is developed with the help of an assumed-mode technique. This model is solved by means of the pseudo-arclength continuation technique and a direct time integration method based on the modified Rosenbrock scheme. Moreover, a linear analysis is performed to determine the natural frequencies of the linear system. The effect of the small-size parameter on the principal parametric response of the system is analysed. The detailed dynamics of the system is examined by plotting the parametric frequency-responses, parametric force-responses, time histories, phase-plane diagrams, fast Fourier transforms (FFTs), and Poincaré sections.

2. Formulations for kinetic and potential energies

The schematic representation of the system under consideration is shown in Fig. 1, showing a geometrically imperfect microbeam of length L, thickness h, flexural stiffness EI, axial stiffness EA, and mass density ρ. The microbeam is hinged at both ends and a time-dependent axial load $P_0 + P_1 \cos(\omega t)$ is exerted in the axial direction. x and z are the axial and transverse coordinates, respectively; $u(x, t)$ and $w(x, t)$ represent the displacements in the longitudinal and transverse directions, respectively—t is time. $w_0(x)$ is the arbitrary initial curvature in the transverse direction.

The equations of motion are derived under the following assumptions: (i) the Euler–Bernoulli beam theory is used, neglecting the effect of shear deformation and rotary inertia; (ii) the cross-section of the beam is constant along the entire length; (iii) the nonlinearity arises from the mid-plane stretching of the microbeam; (iv) the final assumption is that there is no warping in the system.