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Heterogeneous perturbation of fluid density and solid elastic strain in consolidating porous media

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ABSTRACT

The occurrence of heterogeneous perturbations of fluid mass density and solid elastic strain of a porous continuum, as a consequence of its undrained response is a very important topic in theoretical and applied poromechanics. The classical Mandel-Cryer effect provides an explanation of fluid overpressure in the central region of a porous sample, immediately after the application of the loading. However this effect fades away when the fluid leaks out of the porous network. Here this problem is studied within the framework of a second gradient theory and a thorough description of the static and the dynamics of the phenomenon is given. We study how the presence of an impermeable wall affects the formation of the interface between two phases differing in the fluid content. Moreover, we show that the late time interface motion towards its stationary position is not affected by the impermeable wall and is characterized by a common seepage velocity profile.

1. Introduction

When a mechanical pressure is exerted on the solid skeleton of a porous medium and its elastic strain is a consequence of the variation of the fluid mass content inside the pores, several interesting phenomena can occur which accompany shrinkage or swelling of the solid skeleton. The focus, here, is on the occurrence of heterogeneous perturbations of the fluid mass density and the skeleton elastic strains, as a consequence of the undrained response of the porous medium. The classical Mandel-Cryer effect, see (Mandel, 1953) and (Cryer, 1963), provides an explanation, within multidimensional consolidation, of fluid overpressure, in the central region of the sample, immediately after the application of the loading. However, in that case this is a definitely non-permanent effect which fades away when the fluid leaks out the boundary and the pore pressure reverses and dissipates. The physical background of the Mandel-Cryer effect is that the generation of fluid over-pressure due to loading is immediate, but the dissipation due to the fluid flow is retarded by the permeability and the distance to the drainage boundary. On the other hand several authors discussed the onset of strain localization during globally undrained triaxial tests, in particular for loose granular materials, see e.g., Mokni and Desrues (1998); Mooney, Viggiani, and Finno (1997), or Sulem and Ouffroukh (2006). In this case local fluid exchange is allowed, even in presence of localized strain, inside the specimen until, at high level of confinement, the pore pressure generation inside the band leads locally to fluidization of the crushed material, which results into the formation of connected channels in the heart of the band. Similar confinement effects have also been recorded in a fluidized column test, see (Nichols, Sparks, & Wilson, 1994), where a fluid is forced to flow through a saturated sample from the bottom. By tuning the

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velocity of the fluid, the drag force acting on the solid grains, possibly causing unbalance of gravity force, is controlled (Vardoulakis, 2004a; 2004b). These experimental results demonstrate that under consolidation loading, and because of porosity change, the fluid can migrate through the pores and eventually remain segregated, possibly enhancing localized overpressurization and fluidization of the soil, see e.g. Kolymbas (1994); Nichols et al. (1994). These porosity modulations have been observed in Holcomb and Olsson (2003); Olsson and Holcomb (2000), during the consolidation process of a sandstone, and in Lenoir, Andrade, Sun, and Rudnicki (2010) at stationarity.

In the papers Cirillo, Ianiro, and Sciarra (2009, 2010, 2011) and Cirillo, Ianiro, and Sciarra (2013) we have attacked this problem from the point of view of bifurcation theory and we have shown that it is possible to describe interesting phenomena (still in the range of non-linear elasticity) taking place when the confining pressure exerted on the solid exceeds a suitable limiting value. The idea is to get a formulation capable for describing the onset of a fluid-rich and a fluid-poor phase, eventually coexisting inside the porous skeleton at equilibrium. Introducing a non-local energy contribution, which penalizes gradients of strain and fluid mass density, a smooth transition between phases of the porous medium, associated with different fluid content, has been modeled, so accounting for the arising of the above mentioned heterogeneous elastic strains. Undrained conditions are therefore locally achieved where fluid segregation is attained, even if a standard Darcean dissipative process, associated to the fluid flowing out of the drainage boundary, occurs.

Assuming the potential energy to be quadratic in the first derivatives of the strain and of the fluid mass density variation, the evolution is described by a Cahn-Hilliard-like equation provided that the dissipative forces are proportional to the seepage velocity, say the velocity of the fluid with respect to the solid. This means that the above mentioned assumption of Darcean flow still remains valid.

Within this modeling framework, a generalized consolidation problem for a one-dimensional porous continuum is analyzed so extending the classical results due to Terzaghi and those ones, relative to a gradient model, previously obtained by one of the authors, and coworkers (Sciarra, Dell-Isola, Ianiro, & Madeo, 2008), in which only the fluid-poor phase was admissible at equilibrium. The equation governing the behavior of the fluid constituent is of higher (fourth) order with respect to the Laplace equation which classically prescribes the behavior of the pore-water pressure. Following previous results reported in Cirillo et al. (2013), different boundary conditions can be considered, in particular essential or natural boundary conditions on the velocity of the fluid relative to the solid and on the fluid chemical potential as well as on the fluid mass density or on its spatial gradient. The dependence of the boundary value problem on higher order derivatives has been taken into account. Here we shall address the two cases in which zero chemical potential, see Section 2.4, or zero fluid velocity, say impermeability of the porous skeleton, see Section 2.5, have been assumed on the whole or part of the boundary, together with essential boundary conditions on the strain of the solid and the density of the fluid. As already mentioned the interest will be in the occurrence of heterogeneous elastic strains of the solid skeleton and variations of the fluid density; the confining pressure is therefore chosen so as to guarantee the coexistence of phases and, consequently, the onset and the propagation, up to its stationary placement, of the interface between them, see (Cirillo, Ianiro, & Sciarra, 2012). In these two cases we describe the formation of the interface between the phases and its motion towards its stationary location. In particular, we show that the late time interface motion towards its stationary position is not affected by the impermeable wall and is characterized by a common seepage velocity profile.

2. The model

We introduce the one dimensional poromechanical model (Cirillo et al., 2013) whose geometrically linearized version will be studied in the following sections. Kinematics will be briefly resumed starting from the general statement of the model (Coussy, 2004) together with some particular issue introduced in Sciarra et al. (2008). The equations governing the behavior of the porous system will then be deduced prescribing the conservative part of the constitutive law through a suitable potential energy density Φ and the dissipative contributions through purely Darcy terms.

2.1. Poromechanics setup

Let $B_s := [\ell_1, \ell_2] \subset \mathbb{R}$, with $\ell_1, \ell_2 \in \mathbb{R}$, and $B_f := \mathbb{R}$ be the *reference* configurations for the solid and fluid components (Coussy, 2004). The *solid placement* $\chi_s : B_s \times \mathbb{R} \to \mathbb{R}$ is a C^2 function such that the map $\chi_s(\,\cdot, t)$, associating to each $X_s \in B_s$ the position occupied at time t by the particle labeled by X_s in the reference configuration B_s, is a C^2-diffeomorphism. The *fluid placement* map $\chi_f : B_f \times \mathbb{R} \to \mathbb{R}$ is defined analogously. The *current configuration* $B_t := \chi_s(B_s, t)$ at time t is the set of positions of the superposed solid and fluid particles.

Consider the C^2 function $\phi : B_s \times \mathbb{R} \to B_f$ such that $\phi(X_s, t)$ is the fluid particle that at time t occupies the same position of the solid particle X_s; assume, also, that $\phi(\,\cdot, t)$ is a C^2-diffeomorphism mapping univocally a solid particle into a fluid one. The three fields[1] χ_s, χ_f, and ϕ are not at all independent; indeed, by definition, we immediately have that $\chi_f(\phi(X_s, t), t) = \chi_s(X_s, t)$ for any $X_s \in B_s$ and $t \in \mathbb{R}$.

The Lagrangian velocities are two maps associating with each time and each point in the solid and fluid reference space the velocities of the corresponding solid and fluid particles at the specified time. More precisely, the *Lagrangian velocities* are the two

[1] In the sequel we shall often use the inverse functions of the field χ_f, χ_s, and ϕ with respect to the solid and fluid reference configuration. We shall misuse the notation and let $\phi^{-1}(\cdot, t)$ be the inverse of the map $X_s \to \phi(X_s, t)$ at a given time t. Similarly we shall also consider $\chi_s^{-1}(\cdot, t)$ and $\chi_f^{-1}(\cdot, t)$.