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Surface stress effects on the nonlinear postbuckling characteristics of geometrically imperfect cylindrical nanoshells subjected to axial compression

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ABSTRACT

For structures at nanoscale, the surface effects can be important due to the high ratio of surface area to volume. In the current investigation, the nonlinear axial postbuckling behavior of geometrically imperfect cylindrical nanoshells is studied including surface stress effects. For this purpose, Gurtin–Murdoch continuum elasticity theory in conjunction with von Karman–Donnell-type geometric nonlinearity is implemented into the classical shell theory. By the developed size-dependent shell model, the surface effects which include surface elasticity and residual surface stress are taken into account. In order to satisfy balance conditions on the surfaces of nanoshell, a linear variation through the thickness is considered for the normal stress component of the bulk. Based on the variational approach using virtual work's principle, the non-classical governing differential equations are derived. In order to solve the nonlinear problem, a boundary layer theory is employed which contains simultaneously the nonlinear prebuckling deformations, initial geometric imperfections and large deflections corresponding to the postbuckling domain. Subsequently, a two-stepped singular perturbation methodology is utilized to predict the nonlinear critical buckling loads as well as the postbuckling equilibrium paths. It is observed that by taking surface stress effects into account, the both critical buckling load and critical end-shortening of a cylindrical nanoshell made of Silicon increase.

1. Introduction

The importance of understanding the nanoscale mechanics to fabricate prosperous design of various nanostructures and nanodevices is increasing with the growth of interest in nanoscience and nanotechnology. Approaching the physical dimensions of a structure to the nanoscale makes its mechanical properties be size-dependent. The surface stress effect is one of the significant molecular effects which can be easily observed at the atomic scale, and this has been clearly indicated and explained (Dingreville, Qu, & Cherkaoui, 2005; Streitz, Cammarata, & Sieradzki, 1994). Due to the different environment conditions, atoms at or near a free surface have different equilibrium requirements than the atoms have in the bulk of the material. This difference leads to excess surface energy as a superficial energy term since a surface can be interpreted as a layer to which certain energy is attached (Fischer, Waitz, Vollath, & Simha, 2008).

Because of difficulties encountered in the experimental methods to predict the responses of nanostructures under different loading conditions as the size of physical systems is scaled down into the nanoscale, theoretical analyses have been more

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noteworthy. The classical continuum mechanics has been widely used to study the responses of nanostructures (Ansari, Hemmatnezhad, & Rezapour, 2011; Fu, Hong, & Wang, 2006; Liew & Wang, 2007; Wang, Ni, Li, & Qian, 2008; Yoon, Ru, & Mioduchowski, 2003). However, it does not have the capability to apply directly in the analysis of nanoscale domains due to scale independency. In order to study the size-dependent response of structures at nanoscale, modified continuum models are utilized as one of the most applied theoretical approaches because of their computational efficiency and the capability to produce accurate results which are comparable to those of atomistic models (Ansari & Sahmani, 2012; Ansari & Sahmani, 2013; Ansari, Gholami, & Sahmani, 2011; Ansari, Sahmani, & Arash, 2010; Ansari, Sahmani, & Rouhi, 2011; Hu, Liew, Wang, He, & Yakobson, 2008; Sahmani & Ansari, 2013; Sahmani & Ansari, 2013; Sahmani, Ansari, Gholami, & Darvizeh, 2013; Sahmani, Bahrami, & Ansari, 2014; Shen, 2013; Shen & Zhang, 2010; Wang, Zhao, & Zhou, 2010).

A theoretical framework based on the continuum mechanics incorporating surface stress effects was proposed by Gurtin and Murdoch (Gurtin & Murdoch, 1975; Gurtin & Murdoch, 1978). On the basis of this continuum elasticity theory, the surface of structure is simulated as a mathematical layer with zero thickness containing different material properties from the underlying bulk which is completely bonded by the membrane. This point should be noted that in order to achieve accurate results, some special parameters extracted from interatomic potentials or atomistic properties should be incorporated into the modified continuum mechanics model. In the case of Gurtin–Murdoch elasticity theory, by using the proper values of surface elastic constants (without any limitation in the value of thickness of surface), the obtained results may have a good agreement with those of atomistic models. For example, Miller and Shenoy, 2000 computed surface moduli of different surface orientations by using the embedded atom method for FCC Aluminum and Stillinger–Weber empirical potentials for Silicon and they demonstrated that the size-dependent behavior of nanoscale structural elements can be modeled by applying the Gurtin–Murdoch continuum model including surface stress effects, whose results are almost indistinguishable from the atomistic simulations for nanoplates.

In recent years, several investigations have been carried out which confirm the excellent capability of Gurtin–Murdoch elasticity theory to consider surface effects in different mechanical responses of nanostructures. For instance, He et al. (Lu, He, & Lee, 2006) proposed a continuum model based on Gurtin–Murdoch elasticity theory for size-dependent deformation of elastic films of nanoscale thickness. Li et al. (Li, Lim, & He, 2006) studied the influence of surface effect on stress concentration around a spherical cavity in a linearly isotropic elastic medium on the basis of continuum surface elasticity. Mogilevskaya et al. (Mogilevskaya, Crouch, & Stolarski, 2008) considered a two-dimensional problem of multiple interacting circular nano-inhomogeneities and nano-pores based on Gurtin–Murdoch model. Luo and Wang, 2009 investigated the elastic field of an elliptic nano inhomogeneity embedded in an infinite matrix under anti-plane shear. Park, 2009 demonstrated the influence of surface residual stress on the resonant frequencies of Silicon nanowires using surface elasticity theory. The interface stress effects of the nano inhomogeneity were accounted for with Gurtin–Murdoch model. Ansari and Sahmani, 2011 used Gurtin–Murdoch elasticity theory to predict the bending and buckling behaviors of nanobeams. They performed an analytical solution to obtain explicit formulas for critical buckling of nanobeams in the presence of surface effects. Gheshlaghi and Hasheminejad, 2011 examined the nonlinear flexural vibrations of simply supported Euler–Bernoulli nanobeams via an exact solution method with consideration of surface stress effect. The free vibration characteristics of rectangular nanoplates including surface stress effect were investigated by Ansari and Sahmani, 2011. They implemented the Gurtin–Murdoch elasticity theory into the classical first-order shear deformation plate theory to capture size effect. Recently, Ansari et al. (Ansari, Mohammadi, Faghih Shojaei, Gholami, & Sahmani, 2013, 2014) predicted the postbuckling characteristics of nanobeams in the presence of surface stress by using Gurtin–Murdoch elasticity theory within the framework of Euler–Bernoulli and Timoshenko beam theories, respectively.

Recently, based on the surface elasticity theory, Wang and Wang, 2014 studied the influence of surface effects on the postbuckling behavior of rectangular nanoplates. Sahmani et al. (Sahmani, Bahrami, & Ansari, 2014) examined the free vibration behavior of third-order shear deformable nanobeams in the vicinity of postbuckling configuration and in the presence of surface effects. They also investigated the free vibration characteristics of circular higher-order shear deformable nanoplates around the postbuckling configuration incorporating surface effects (Sahmani, Bahrami, & Ansari, 2014). Sahmani et al. (Sahmani, Bahrami, Aghdam, & Ansari, 2014) predicted the nonlinear forced vibration response of third-order shear deformable nanobeams with the presence of surface effects. Wang and Wang, 2015 developed a general model for nano-cantilever switches with consideration surface effects, nonlinear curvature and the location of fixed electrode. Sahmani et al. (Sahmani, Aghdam, & Bahrami, 2015) investigated the free vibration response of third-order shear deformable nanobeams made of functionally graded materials around the postbuckling domain.

In the present investigation, Gurtin–Murdoch elasticity theory is implemented into the classical shell theory with von Karman–Donnell-type of kinematic nonlinearity to develop non-classical shell model incorporating surface stress effects. Then by using the size-dependent shell model, the nonlinear postbuckling behavior of geometrically imperfect cylindrical nanoshells subjected to axial compression is predicted with the presence of surface stress effects. Afterwards, on the basis of a boundary layer theory, a two-stepped singular perturbation technique is put to use in order to obtain the nonlinear critical buckling loads and postbuckling equilibrium paths of cylindrical nanoshells including nonlinear prebuckling deformations, initial geometric imperfections and surface stress effects.

2. Preliminaries

A cylindrical nanoshell with the length L, thickness h, and mid-surface radius R is considered as shown in Fig. 1. The nanoshell includes a bulk part and two additional thin surface layers (inner and outer layers). For the bulk part, the material properties are Young's modulus E and Poisson's ratio ν. The two surface layers are assumed to have surface elasticity modulus of E_s, Poisson's