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A unified theoretical structure for modeling interstitial growth and muscle activation in soft tissues

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ABSTRACT

The objective of this paper is to develop a new unified theoretical structure for modeling interstitial growth and muscle activation in soft tissues. The model assumes a simple continuum with a single velocity field. In contrast with many other formulations, evolution equations are proposed directly for a scalar measure of elastic dilatation and a tensorial measure of elastic distortional deformation. The evolution equation for elastic dilatation includes a rate of mass supply or removal that controls volumetric growth and causes the elastic dilatation to evolve towards its homeostatic value. Similarly, the evolution equation for elastic distortional deformation includes a rate of growth that causes the elastic distortional deformation tensor to evolve towards its homeostatic value. Specific forms for these inelastic rates of growth and the associated homeostatic values have been considered for volumetric, area and fiber growth processes, as well as for muscle activation. Since the rate of growth appears in the evolution equations and not a growth tensor it is possible to model the combined effects of multiple growth and muscle activation mechanisms simultaneously. Also, robust, strongly objective, numerical algorithms have been developed to integrate the evolution equations.

1. Introduction

Biological tissues are complicated materials which are mixtures of many components that can flow relative to each other and interact mechanically, chemically and electrically (e.g. Humphrey & Rajagopal, 2002; Ateshian, Costa, Azeloglu, Morrison, & Hung, 2009; Ambrosi et al., 2011; Ateshian, Morrison, Holmes, & Huang, 2012; Sciume et al., 2013). From the point of view of continuum mechanics it is natural to model these tissues using mixture theory (e.g. Green & Naghdi, 1965, 1967; Ateshian & Humphrey, 2012). However, complications of multiple flow fields and constitutive equations for mechanical, biochemical and electrical coupling of the components in the mixtures make progress using this approach slow. Also, the numerical implementation of these equations for the large deformations experienced by soft biological tissues is challenging.

The equations of mixture theory can be simplified considerably by assuming that all components move with the same velocity field. An example of such a mixture theory, where the components can be produced and removed potentially in different stressed configurations, is given in Humphrey and Rajagopal (2002). They discussed a number of critical ingredients

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needed for a phenomenological theory of growth but they only presented an outline of possible constitutive equations. Within the context of this approximation, the composite material can be treated as a simple continuum with constitutive equations for the individual components that can interact or with a constitutive equation for a homogenized mixture that attempts to model the main responses of the tissue. Assuming that the velocity field and its spatial gradient are continuous and bounded and using the fact that a material point has a single velocity, it is possible to define a one-to-one mapping between the position of a material point in any reference configuration and its current position. Consequently, it is possible to define a material region which contains the same material points for all time. Moreover, in this approach the material region can still be modeled as an open system in the sense that mass can be supplied or removed at each material point. More complicated motions of growing materials have been discussed by Cowin (2010).

Taber (1995) presented a review of the literature related to growth, remodeling and morphogenesis of biological tissues. He connected growth with change in mass, remodeling with change in material properties and morphogenesis with change in shape. He also stated that these processes are linked in general but are usually treated separately. Hsu (1968) is one of the first researchers to study the influence of mechanical loads on growth, which was modeled using both an external volume mass supply term and a mass flux that controls mass diffusion. Moreover he used the analogy of creep in metals and referred to growth as "a slow deviation of the body from its original form...". Within the context of thermodynamics with chemical reactions, Cowin and Hegedus (1978) developed a finite deformation theory of elasticity with growth and remodeling of bone due to mass changes. Although this theory was developed for finite deformations, bones typically experience only small strains before they fracture. In contrast, soft tissues truly experience large deformations so it is important that models for soft tissues be developed within the context of a finite deformation theory. Skalak (1981) and Skalak et al. (1982) presented a finite deformation theory of volumetric and surface growth of biological tissues. Cowin (1986) studied remodeling of bone within the context of small deformation anisotropic elasticity in which the stress tensor was an isotropic function of the strain tensor and a fabric tensor. In this theory the stress vanishes when the strain vanishes so that growth and morphogenesis cannot be modeled.

As mentioned previously, Hsu (1968) made connections between growth and plasticity of metals. Rodriguez, Hoger, and McCulloch (1994) developed equations for finite deformation growth of soft tissues which have a similar structure to those used to model plasticity of metals. These equations can model the changing shape of the material in its unstressed intermediate configuration. Lubarda and Hoger (2002) considered a generalized finite deformation theory of growth. They developed constitutive equations within the context of a thermomechanical theory with chemical energy transfer. In both of these works (Rodriguez et al., 1994; Lubarda & Hoger, 2002) it was assumed that the stress $\mathbf{T}$ depends on an elastic deformation tensor $\mathbf{F}_e$, which can be defined in terms of the total deformation gradient $\mathbf{F}$ and a growth deformation tensor $\mathbf{F}_g$ by the multiplicative form

$$\mathbf{F}_e = \mathbf{F}\mathbf{F}_g^{-1}. \tag{1.1}$$

The growth tensor $\mathbf{F}_g$ is determined by integrating an evolution equation

$$\dot{\mathbf{F}}_g = \mathbf{\Lambda}_g \mathbf{F}_g, \tag{1.2}$$

where a superposed $(\dot{\cdot})$ denotes material time differentiation, $\mathbf{\Lambda}_g$ requires a constitutive equation and in this text no sum is implied on repeated indices (e, g, p, h). Furthermore, using the definition of the velocity gradient $\mathbf{L}$ and the derivative of the inverse of $\mathbf{F}_g$

$$\mathbf{L} = \dot{\mathbf{F}}\mathbf{F}^{-1}, \quad \frac{d}{dt}\left(\mathbf{F}_g^{-1}\right) = -\mathbf{F}_g^{-1}\mathbf{\Lambda}_g, \tag{1.3}$$

it can be shown that the elastic deformation tensor $\mathbf{F}_e$ satisfies the evolution equation

$$\dot{\mathbf{F}}_e = (\mathbf{L} - \mathbf{L}_g)\mathbf{F}_e, \tag{1.4}$$

where $\mathbf{L}_g$ takes the form

$$\mathbf{L}_g = \mathbf{F}_e \mathbf{\Lambda}_g \mathbf{F}_e^{-1}. \tag{1.5}$$

Eq. (1.4) has the same form as the evolution equation introduced by Besseling (1966) for characterizing the rheology of elastically anisotropic elastic–plastic materials.

Lubarda and Hoger (2002) proposed elastically orthotropic constitutive equations in which the Helmholtz free energy ψ (per unit mass) depended on invariants of the deformation tensor $\mathbf{C}_e$ defined by

$$\mathbf{C}_e = \mathbf{F}_e^T \mathbf{F}_e, \tag{1.6}$$

and two structural tensors. Moreover, the functional form for ψ was restricted so that the material is stress-free in its intermediate configuration with

$$\mathbf{T} = 0 \quad \text{for} \quad \mathbf{C}_e = \mathbf{I}, \tag{1.7}$$

which means that stress vanishes when $\mathbf{F}_e$ is equal to a proper orthogonal rotation tensor.