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Nonlinear effects in composite cylinders: relations and dependence on inhomogeneities

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ABSTRACT

The extension of a cylinder subjected to torsion was investigated by Poynting (1909) and the nonlinear phenomenon has since been called the Poynting effect. Under combined axial and torsional loading, the twist is also dependent on the axial loading as shown by Wang and Wu (2014a), who named this the axial force–twist effect. This paper investigates the relations between these effects in a bilayered cylindrical composite within the framework of second-order elasticity. The results show that: (1) either effect can be positive or negative, (2) there can only be three states, e.g., under tension–torsion either both effects are negative, or both positive, or if they differ in sign the Poynting effect must be positive and the axial force–twist effect must be negative, (3) certain logical relations between the effects exist, e.g., if under tension–torsion loading the Poynting effect is negative, the axial force–twist effect must be negative, (4) there exists a reduced elastic-geometric parameter between each effect and the associated applied loading, and (5) both effects are strong functions of the elastic and geometric inhomogeneities. These findings are significant for applications in regenerative medicine such as the design of replacement tissues.

1. Introduction

The extension of a cylinder under simple torsion was discovered by Poynting (1909) in the last century. This nonlinear phenomenon, accordingly named the Poynting effect, was experimentally observed for steel, copper and brass wires (Poynting, 1909, 1912) as well as for rubber-like solids (Lenoe, Heller, & Freudenthal, 1965). Nearly a century after its discovery, it was found that biopolymer gels sheared between parallel plates tend to draw the plates together, or they tend to develop a negative normal stress which may be comparable in magnitude to the shear stress (Janmey et al., 2007; Storm, Pastore, MacKintosh, Lubensky, & Janmey, 2005). This negative normal stress is akin to the axial contraction of a cylinder subjected to torsion, i.e., it corresponds to the negative or reverse Poynting effect. Experimental results supporting the occurrence of the negative effect in twisted fibers are available, e.g., stretched rabbit papillary muscles contracted in length when twisted (Horgan & Murphy, 2012).

To understand the Poynting effect, nonlinear elastic models have been used to investigate the torsion of bars and the shear of rectangular blocks. The strain energy functions can take various forms such as the neo-Hookean, Mooney-Rivlin, Ogden, and polynomial/exponential. Recent studies include (Destrade & Saccomandi, 2010; Horgan & Murphy, 2011, 2012; Mihai & Goriely, 2011, 2013; Wang & Wu, 2014a, 2014b; Wu & Kirchner, 2010;). Among these are the studies of

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the effect in transversely isotropic materials (Horgan & Murphy, 2011, 2012) and of bilayered composites (Wang & Wu, 2014a, 2014b). On the basis of second-order isotropic elasticity, Wang and Wu (2014a) demonstrated the possibility of both the Poynting effect under pure torsion and the "axial force–twist" effect under combined axial–torsional loading. The latter refers to a coupled nonlinear phenomenon in which the twist of a cylinder is affected by the axial loading, in addition to the applied torsion. The axial force–twist effect can also be either positive or negative. The positive effect means that the axial loading (either tension or compression) enhances the twist, while the negative effect means that it reduces the twist. The existence of such an effect is also suggested in the theoretical work of Zubov (2001), who referred this as the "inverse Poynting effect".

Nonlinear effects can play a pivotal role in the design considerations of artificial materials and in the understanding of biological functions, in view of much evidence that such functions can be significantly affected by mechanical stresses and deformations. For instance, the negative normal stress can influence the movement of mitochondria through the cytoskeleton of a narrow axon without distending the axon diameter, and it may also help in the compression of a fibrin gel at a wound site to a blood vessel due to shear flow in the vessel (Janmey et al., 2007). The interaction between surgical tool and tissue often occurs in the shearing mode, and the Poynting effect may result in normal forces that are larger or smaller than the human perception threshold for force discrimination (Misra, Ramesh, & Okamura, 2010). This has important implications for the design of surgical simulation systems that provide haptic feedback.

Besides nonlinearity and finite deformation, inhomogeneity, anisotropy and viscoelasticity may also play an important role in tissue mechanics. Inhomogeneity is given the primary focus in this work, as many biological and technological materials are composites. For instance, the aortic valve leaflet is composed of three morphologically distinct layers: fibrosa, spongiosa and ventricularis, and the development of novel heart valve therapies would depend on the synergistic mechanical behavior of the composite (Stella & Sacks, 2007). Another example is the development of polyelectrolyte multilayered capsules for the delivery of stem cells to induce bone formation in vivo (Facca et al., 2010). The multilayered structure enables the controlled delivery of different active molecules aimed at therapies such as gene and drug therapy.

Previous theoretical works have dealt largely with the Poynting effect in homogeneous materials. Wang and Wu (2014a) studied both the Poynting effect and the axial force–twist effect in a bilayered cylindrical composite. They showed that analytical expressions determining the sign and magnitude of the effects can be obtained for a bilayered cylindrical composite. However, the relations between these two effects, and their dependence on the elastic and geometric inhomogeneity between the layers, has not been explored. Elastic inhomogeneity refers to the dissimilarity between the layer elastic constants (both second- and third-order). Geometric inhomogeneity refers to the possible varying layer sizes: if r_1 and r_2 denote respectively the outer radii of the core and the outer layer of a bilayered cylinder, then $r_1 = 0$ and $r_1 = r_2$ both denote homogeneous cylinders while the ratio $r_{1/}r_2$ would serve as a measure of geometric inhomogeneity. This paper attempts to demonstrate that certain universal relations can be obtained for the Poynting and the axial force–twist effect of homogeneous, bilayered as well as hollow cylinders. Furthermore, it is shown that these effects are effectively captured by single parameters which are special combinations of the elastic and geometric parameters. The size dependence of these effects is also revealed in the analytical expressions. Investigation is carried out to show numerically how these effects depend on the elastic and geometric inhomogeneities.

This paper is organized in the following manner. A brief recapitulation of the second-order elastic model is first presented in Section 2, which incorporates analytical results for characterizing the Poynting effect and the axial force–twist effect in a bilayered cylinder. In particular, a universal relation between the two effects is demonstrated. The dependence of these effects on the elastic and geometric inhomogeneities is then presented in some detail in Section 3, highlighting the controllability of both the magnitude and sign of the effects. Further discussion follows in Section 4, and the paper concludes with a summary in Section 5.

2. Analytical expressions for the Poynting and axial force–twist effect

A summary of the formulation is given in Sections 2.1 and 2.2 below; the details are available in Wang and Wu (2014a). Based on the most general inhomogeneous case, further consideration of the following cases follows in Section 2.3: linearly inhomogeneous but nonlinearly homogeneous, linearly homogeneous but nonlinearly inhomogeneous, homogeneous, and hollow homogeneous. The emphasis of the current paper is on the relations between the Poynting effect and the axial force–twist effect for these various cases, as explained in Section 2.4.

2.1. Governing equations

The physical problem is that of a cylinder with two dissimilar concentric layers, as shown in Fig. 1. The core has a radius of r_1 while the outer layer has inner and outer radii of r_1 and r_2, respectively. The second-order elastic constants are the usual Lamé constants denoted by λ_i, μ_i, while the third-order ones by l_i, m_i and n_i, where i = 1, 2 denote the inner and outer layers, respectively. The bilayered cylinder is subjected to combined torsion T and axial loading P. The formulation below follows the perturbation procedure of Murnaghan (1951).

The final coordinates (ρ, ψ, ζ) of the particle of the cylinder whose initial coordinates being $(r,\ \theta, z)$ are assumed to be $(r + u_r,\ \theta + u_\theta/r,\ z + u_z)$, where the radial displacement $u_r = kF(r)$, the angular displacement $u_\theta/r = kG(z)$ and the axial