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# Modified nonlocal elasticity theory for functionally graded materials

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ABSTRACT

In this paper, it will be shown that the nonlocal theory of Eringen is not generally suitable for analysis of functionally graded (FG) materials at micro/nano scale and should be modified. In the current work, an imaginary nonlocal strain tensor is introduced and used to directly obtain the nonlocal stress tensor. Similar to the stress tensor in Eringen's nonlocal theory, the imaginary nonlocal strain tensor at a point is assumed to be a function of local strain tensor at all neighbor points. To compare the new modified nonlocal theory with Eringen's theory, free vibration of FG rectangular micro/nanoplates with simply supported boundary conditions are investigated based on the first-order plate theory and three-dimensional (3-D) elasticity theory. The material properties are assumed to be functionally graded only along the plate thickness. The effects of nonlocal parameter and material gradient index on the natural frequencies of FG micro/nano plates are discussed. The present developed nonlocal theory can be used in conjunction with different analytical and numerical methods to analyze mechanical response of micro/nano structures made of FG materials.



## 1. Introduction

Functionally graded materials are an advanced class of composite materials where the volume fractions of the constitutive materials are varied continuously as a function of position from one point to the other. This continuity provides continuous distribution of material properties and results in eliminating common interface difficulties of laminate composite materials (Koizumi, 1993). The advanced physical and mechanical characteristics of FG materials make them to be research matter for diverse disciplines as tribology, geology, optoelectronics, biomechanics, fracture mechanics and micro- and nanotechnology (Suresh & Mortensen, 1998; Suresh, 2001). Introducing of FG materials to micro- and nanotechnology has led to easily achieving the micro/nano devices, with better physical properties, such as micro/nano electromechanical systems (Witvrouw & Mehta, 2005; Lee et al., 2006), shape memory alloy thin films (Fu, Du, & Zhang, 2003) and atomic force microscopes (Rahaeifard, Kahrobaiyan, & Ahmadian, 2009).

The size dependence of mechanical behavior in micro/nano scale makes the applicability of classic continuum theory somewhat questionable. The classic continuum theory which is scale independent does not capture small size effect, and so, cannot correctly predict mechanical behavior of micro/nano structures. Hence, non-classical continuum theories such as classical couple stress (Toupin, 1962; Mindlin & Tiersten, 1962; Mindlin, 1963; Koiter, 1964), strain gradient (Aifantis,

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1999), nonlocal elasticity (Eringen, 1972, 1972) and modified couple stress (Yang, Chong, Lam, & Tong, 2002) have been extended to accommodate the size effect of micro/nanomaterials. In the classical couple stress theories (Toupin, 1962; Mindlin & Tiersten, 1962; Mindlin, 1963; Koiter, 1964), the material particle can be under applied loads including not only a force to translate the material particle but also a couple to rotate it. Based on this concept, strain energy is a function of both strain and curvature tensors. Recently, the modified couple stress theory has been developed by Yang et al. (2002). The two main differences between the modified couple stress theory and the classical couple stress theory are the involvement of only one material length scale parameter and that the strain energy is a function of the strain and only the symmetric part of the curvature tensor. In the strain gradient theory (Aifantis, 1999), it is assumed that the strain energy function depends on the gradient of the strain tensor in addition to its strain tensor. The strain gradient theory includes three length scale parameters for the dilatation gradient vector, the curvature tensor and the deviatoric stretch gradient tensor. In the nonlocal theory of Eringen (Eringen, 1972, 1972), the stress at a point in a continuum body is assumed to be a function of the strain at all neighbor points of the domain, because of atomic forces and micro/nano scale effect.

The nonlocal theory of Eringen has been developed for isotropic and homogeneous materials. But, it has been used in many works to study different mechanical behavior of micro/nano structures made of FG materials such as micro/nano beams and plates, see for example (Natarajan, Chakraborty, Thangavel, Bordas, & Rabczuk, 2012; Eltaher, Emam Samir, & Mahmoud, 2012; Simsek, 2012; Ke, Wang, Yang, & Kitipornchai, 2012; Kahrobaiyan, Rahaeifard, Tajalli, & Ahmadian, 2012; Lei, He, Zhang, Gan, & Zeng, 2012; Jung & Han, 2013; Hosseini-Hashemi, Bedroud, & Nazemnezhad, 2013; Eltaher, Emam Samir, & Mahmoud, 2013; Uymaz, 2013; Simsek & Yurtcu, 2013; Rahaeifard, Kahrobaiyan, Ahmadian, & Firoozbakhsh, 2013; Simsek & Reddy, 2013; Rahim Nami & Janghorban, 2014; Nazemnezhad & Hosseini-Hashemi, 2014; Rahmani & Pedram, 2014; Hosseini-Hashemi, Nazemnezhad, & Bedroud, 2014; Daneshmehr, Rajabpoor, & pourdavood, 2014; Eltaher, Khairy, Sadoun, & Omar, 2014; Daneshmehr, Rajabpoor, & Pourdavood, 2014; KE, Yang, Kitipornchai, & Wang, 2014; Akgoz & Civalek, 2014; Salehipour, Nahvi, & Shahidi, 2015). In this paper, it will be illustrated that the theory of Eringen cannot generally be applied to analyze mechanical behavior of FG micro/nano structures and must be modified for such analysis. For this purpose, a nonlocal strain tensor is defined which is similar to the stress tensor in Eringen's theory. The nonlocal strain tensor is directly used to obtain the nonlocal stress tensor. Based on the presented modified theory, free vibration of FG rectangular micro/nanoplates is analytically carried out using the first-order plate theory and three-dimensional elasticity. The plate is assumed to be simply supported at all edges and the material properties are functionally graded only along the plate thickness. Numerical results are presented to compare the results of the modified nonlocal theory with those of Eringen's nonlocal theory. Contrary to that observed in the Eringen's theory, the modified nonlocal theory predicts that by increasing nonlocal parameter and material gradient index, the results of first-order plate theory and 3-D elasticity theory are very close.

## 2. Nonlocal elasticity of Eringen

According to Eringen's nonlocal elasticity, the stress tensor at a point in a continuum body not only depends on the strain tensor at that point but also on the strains at all neighbor points of the continuum body. The constitutive equations for an isotropic homogeneous nonlocal continuum body are expressed as

$$\begin{aligned} \sigma_{ij} &= \int_V \alpha(|x'-x|)t_{ij}(x')dV(x'), \quad \forall x \in V \\ t_{ij} &= C_{ijkl}\varepsilon_{kl} \end{aligned} \tag{1}$$

where $\sigma_{ij}$ and $t_{ij}$ are the nonlocal and local stress tensors, respectively; $\varepsilon_{ij}$ is the strain tensor and $C_{ijkl}$ is the fourth order elasticity tensor. The nonlocal kernel function $\alpha(|x'-x|)$ incorporates into the constitutive equations the nonlocal effects of local stress at the source point $x'$. Function $\alpha(|x'-x|)$ depends on $\tau = e_0a/l$, in which $e_0$ is material constant, and $a$ and $l$ are internal and external characteristic lengths. Following experimental observations, Eringen (1972) proposed the following forms of function $\alpha(|x'-x|)$ for 2-D and 3-D problems, respectively:

$$\alpha(|x'-x|) = \left(2\pi l^2\tau^2\right)^{-1} K_0(|x'-x|/l\tau) \tag{2a}$$

$$\alpha(|x'-x|) = \left(\pi l^2\tau\right)^{-3/2} \exp\left(-|x'-x|^2/l^2\tau\right) \tag{2b}$$

in which $K_0$ is the modified Bessel function. The integral form of Eq. (1) cannot be solved easily. Hence, a differential form of the constitutive equations for homogenous materials is proposed by Eringen (1972) as

$$\left(1-\mu\nabla^2\right)\sigma_{ij} = t_{ij} \tag{3}$$

where $\mu = (e_0a)^2$ is nonlocal parameter and $\nabla^2$ is Laplacian operator. Based on Eq. (3), stress at a point of continuum body depends on both strain and second gradient of strain at that point. The convenient differential form of the nonlocal Eq. (3) is broadly used in different 1, 2 and 3 dimensional mechanical problems.