International Journal of Engineering Science 90 (2015) 58–75

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Pull-in instability of multi-phase nanocrystalline silicon beams under distributed electrostatic force

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ARTICLE INFO

Article history:
Received 14 December 2014
Received in revised form 31 January 2015
Accepted 14 February 2015
Available online 4 March 2015

Keywords:
Actuated beams
Nano-structured materials
Nanocrystalline silicon
Pull-in voltage
Modified couple stress theory

ABSTRACT

The effects of the material structure on the pull-in instability of nano-actuated beams made of nanocrystalline silicon (Nc–Si) and subjected to a distributed electrostatic force are investigated. Nc–Si is represented as a multi-phase material composed of nano-sized grains, nano voids, and an amorphous-like interface to consider the effects of the interface, grain size, porosity, and the inhomogeneities surface energies on the elastic properties of the composite material. To this end, a size-dependent micromechanical model is developed for multi-phase materials considering the inhomogeneities surface energy effects. An atomic lattice model is also proposed to estimate the elastic modulus of the interface of NcMs. Due to the intensive decrease in the beam's size, the effects of the grain rotations on the beam strain energy and hence on its rigidity are captured and represented using the modified couple stress theory. Considering all these effects and using Euler–Bernoulli beam theory, the governing equation is derived. A finite difference-based solution is used to determine the pull-in voltage of the actuated beams. A parametric study is then performed to reveal the effects of the porosity, interface, surface energy, and grain rotations on the pull-in instability behavior of actuated nano-beams.

1. Introduction

Due to their potentials as sensitive and high frequency devices for applications in Micro-Electro-Mechanical Systems (MEMS) and Nano-Electro-Mechanical Systems (NEMS), micro/nano structures have attracted considerable attention in the last few decades. These electromechanical systems are usually composed of an elastic conductive micro-beam suspended above a rigid conductive plate and a dielectric medium filling the gap between them. The applied electric voltage between the two electrodes produces an electrostatic force compels the elastic beam to deform the system (Batra, Porfiri, & Spinello, 2006). The electrostatic force depends initially on the deflection of the beam. This force has to be balanced by the restoring force of the beam; otherwise the beam continues to deform causes the electromechanical system to collapse. Indeed, the applied voltage has an upper limit beyond which the electrostatic force outweighs the restoring force and the system collapses. This voltage is called pull-in voltage. The determination of its value is critical in order to design efficient MEMS and NEMS devices. For example, to achieve stable operations and enhance device sensitivity, pull-in instability should be avoided in micro/nano mechanical resonators (Tilmans & Legtenberg, 1994) and micro-mirrors (Hung & Senturia, 1999). In switching applications (Xie, Lee, & Lim, 2003), the designer uses the pull-in phenomenon to control the switch on and

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http://dx.doi.org/10.1016/j.ijengsci.2015.02.002
0020-7225/Published by Elsevier Ltd.

off. In all these applications, determining the pull-in instability is essential in order to satisfy the required device specifications.

Recent studies have shown that the pull-in instability behaviors of electromechanical systems are too closely associated with the size of their actuators. Using the modified couple stress theory (Yang, Chong, Lam, & Tong, 2002), the size-dependent static pull-in behavior of electrostatically-actuated micro-beams has been conducted analytically (Baghani, 2012; Rokni, Seethaler, Milani, Hashemi, & Li, 2013) and numerically (Rahaeifard, Kahrobaiyan, Asghari, & Ahmadian, 2011; Shaat & Mohamed, 2014; Yin, Qian, & Wang, 2011). Furthermore, the size-dependent nonlinear dynamical behavior of electrostatically-actuated micro-beams has been investigated by Ghayesh, Farokhi, and Amabili (2013) using the modified couple stress theory. In addition, the strain gradient theory (Mindlin, 1965) has been exploited to study the pull-in phenomenon of electrostatically-actuated micro-beams by Wang, Zhou, Zhao, and Chen (2011) and Rahaeifard and Ahmadian (2015). These studies have proved that considering size-dependent features of the resonator is essential in studying the pull-in behaviors of MEMS and NEMS devices.

Due to the intensive decrease in the size, resonators are usually made from nanostructured materials, such as nanoparticle composites, nanocrystalline materials, and nanoporous structures or from a single-crystal material. Nanocrystalline materials (NcMs) are polycrystalline materials with grain sizes ranging from 1 to 100 nm. In these materials, large volume fractions, which can attain 40%, of atoms reside in interface regions forming an atom-cloud phase. In addition, as reported by Gleiter (2000), the atomic structure in interface regions is different from the perfect lattice in the interiors of grains which is due to misfits and interactions between adjacent crystallites of random orientations. Comparing to their polycrystalline counterparts, which could be represented as a homogeneous material, NcMs are considered as heterogeneous materials. Hence, to efficiently study the pull-in behavior of MEMS and NEMS devices, researchers should consider the heterogeneity nature of their material structure. In a recent research study, Shaat and Abdelkefi (2015) have demonstrated that the interface and the grain size have a significant effect on the behavior of micro/nano-mechanical resonators made of NcMs for bio-mass sensing and disease diagnosis applications.

In micro/nano elastic continua made from NcMs, grains are likely to rotate and to deform in addition to their rigid translations responding to the applied external forces. For elastic continua with large sizes, these additional degrees of freedom (rotations and deformations of grains) are negligible. However, additional material constants are needed for micro/nano elastic continua to capture the effects of these new degrees of freedom on their behaviors. Therefore, additional deformation measures have to be introduced in the strain energy of the actuator to reflect effects of the newly apparent degrees of freedom of the material grains.

Using one of the non-classical continuum theories, several studies have been developed to determine the behavior of nano-sized tubes (Dai, Wang, Abdelkefi, & Ni, 2015), beams (Gao & Mahmoud, 2014; Mahmoud, Eltaher, Alshorbagy, & Meletis, 2012), and plates (Ke, Wang, Yang, & Kitipornchai, 2012; Shaat, 2015). Mahmoud et al. (2012) and Gao and Mahmoud (2014) used, respectively, the differential nonlocal elasticity theory and the modified couple stress theory to investigate the size-dependent behavior of nano-beams when considering their surface energy effects. To study the size dependent effect on micro/nano plates, Ke et al. (2012) and Shaat (2015) used, respectively, the modified couple stress theory and nonlocal elasticity. To the authors' knowledge, no research study has been performed in the literature to simultaneously investigate the inhomogeneity nature of the material structure and higher-order deformations.

In this research study, considering the inhomogeneities size effects along with the effects of the additional rotations of the material grains, an accurate mathematical model is developed to investigate the impacts of the material structure on the pull-in instability of nanocrystalline actuated-beams subjected to distributed electrostatic force. The beam is expected to be made of nanocrystalline silicon which is treated as a multi-phase composite; nano-grains as one phase and porosities as another inclusion phase in an amorphous-like interface phase. An atomic lattice model is proposed to estimate the elastic properties of the interface phase of the NcM. After that, a size-dependent micromechanical model is investigated for multiphase materials in which effects of grains and voids surface energies are included. Moreover, the effects of the rigid rotations of the grains on the beam pull-in behavior are incorporated. To this end, additional higher-order deformation measure is included in the strain energy function based on the assumptions of the modified couple stress theory (Yang et al., 2002). The rest of this paper is organized as follows: in Section 2, a size-dependent micromechanical model for multi-phase materials with nano-inclusions considering their surface energy effects is proposed. Then, in Section 3, the effects of the inhomogeneity nature (porosity and interface effects) on the behavior of actuators made of NcMs are investigated where an atomic lattice model is proposed to estimate the Young's modulus of the interface. The grain rigid rotation effects on the static pull-in voltage of actuated beams under distributed electrostatic force are mathematically modeled in Section 4. In Section 5, a finite difference-based solution is derived for the governing equation. In Section 6, numerical results are presented to reflect the effects of the interface, porosity, and grain rigid rotations on the pull-in instability behavior of actuated beams made of nanocrystalline silicon.

2. A size-dependent micromechanical model for multi-phase materials

A multi-phase composite material with spherical multi-inclusions randomly distributed in a matrix phase is considered. The inclusion surface energies are used in the homogenization scheme. To this end, a multi-phase Representative Volume Element (RVE) is proposed in which isotropic inclusions with their distinct surface phases are randomly distributed in an