International Journal of Engineering Science 90 (2015) 76–85

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Computational evaluation of effective stress relaxation behavior of polymer composites

Tian Tang *, Sergio D. Felicelli

Department of Mechanical Engineering, The University of Akron, Akron, OH 44325, United States

A R T I C L E   I N F O

Article history:
Received 7 December 2014
Received in revised form 3 February 2015
Accepted 14 February 2015
Available online 9 March 2015

Keywords:
Viscoelastic behavior
Polymer composite
Micromechanics
VAMUCH
Effective stress relaxation stiffness

A B S T R A C T

This paper presents a micromechanics model to characterize the effective stress relaxation stiffness of polymer composites. The linear viscoelastic behavior of polymer material was modeled by hereditary integral. The proposed model was established based on the variational asymptotic method for unit cell homogenization (VAMUCH). All computations with this model were accomplished in the time domain, hence the Laplace transform and inversion commonly used for linear viscoelastic composites are not needed in this theory. The accuracy and efficiency of the proposed model were verified by comparing with the results and utilization of finite element models developed using ABAQUS.

1. Introduction

Polymer matrix composites, which are composed of a variety of short or long fibers bound together by organic polymer matrix, have been widely utilized in many engineering areas. Due to the viscoelastic behavior of the polymer matrix, polymer matrix composites exhibit evidently viscoelastic behavior, in which the magnitude of stress and strain are time and temperature dependent. The viscoelasticity phenomenon of polymer composites results from the long molecular chains of the polymer matrix. The creep and stress relaxation responses of polymer composites seriously restrict the advanced composites structures expected to operate for long period of time on many applications (Barbero, 1994).

Micromechanics models are the major tools to characterize the viscoelastic behavior of polymer composites (Aboudi, 2000; Hashin, 1983; Nemat-Nasser & Hori, 1993). Hashin (1965, 1970, 1970) was the first one developing the correspondence principle, which showed that the effective relaxation and creep functions of viscoelastic heterogeneous media are related to the effective elastic moduli of elastic heterogeneous media by the correspondence principle of the theory of linear viscoelasticity. Park and Schapery (1999) developed an efficient and accurate numerical method of interconversion between linear viscoelastic material functions based on a Prony series representation. Their method is straightforward and applicable to interconversion between modulus and compliance functions in time, frequency, and Laplace transform domains. The most common methodology for characterizing the viscoelastic behavior of polymer composites is to apply the Laplace transform and Laplace inversion, where the correspondence principle was applied (Barbero & Luciano, 1995; Christensen, 1979; Haasemann & Ulbricht, 2009; Megnis, Varna, Allen, & Holmberg, 2001; Li, Gao, & Roy, 2006; Schapery, 1967; Li et al., 2006; Wang & Weng, 1992; Yancey & Pindera, 1990). Brinson and Lin (2003) and Fisher and Brinson (2003) employed the finite element method to analyze the two-phase and three-phase viscoelastic composites in the Laplace transformed

* Corresponding author. Tel.: +1 330 972 7672.
E-mail address: tiantang1991@gmail.com (T. Tang).

http://dx.doi.org/10.1016/j.ijengsci.2015.02.003
0020-7225/

domain, respectively, and verified the results with the Mori–Tanaka model (Benveniste, 1987; Mori & Tanaka, 1973). Levin and Sevostianov (2005) proposed an analytical approach for micromechanics modeling of the effective viscoelastic behavior of composites in which the fraction-exponential operator was used to describe the viscoelastic properties of the constituents. Recently, commercial finite element software ABAQUS was applied to predict the stress relaxation response (Abadi, 2009) and creep response (Naik, Abolfathi, Karami, & Ziejewski, 2008) of fiber reinforced polymer matrix composites considering a unit cell subjected to periodic boundary conditions.

The objective of this paper is to develop a micromechanics model to characterize the effective stress relaxation stiffness of viscoelastic polymer composites. The proposed model is an extension of VAMUCH (variational asymptotic method for unit cell homogenization) for effective elastic properties (Yu & Tang, 2007). The effective viscoelastic responses of polymer composites were calculated in time domain. Hence the commonly used Laplace transform and inversion are not required in this theory. Furthermore, the proposed model calculates simultaneously the complete set of effective stress relaxation stiffness so that it is far more convenient than conventional finite element approaches with which multiple running are required on multiple loading and boundary conditions. The accuracy and efficiency of the present modeling technique were validated through the comparison with the results and utilization of finite element models developed based on ABAQUS.

2. Theoretical equations of stress relaxation stiffness of linear viscoelastic materials

Based on the Boltzmann superposition principle, the constitutive equations for the linear viscoelastic material can be expressed in the time domain in the following way,

$$\sigma_{ij}(t) = \int_{-\infty}^{t} C_{ijkl}(t-\tau)\dot{\varepsilon}_{kl}(\tau)d\tau \tag{1}$$

where $C_{ijkl}(t)$ is the stress relaxation stiffness; $\dot{\varepsilon}_{kl}(\tau)$ is the strain rate; $\sigma_{ij}(t)$ is the stress tensor.

The stress relaxation tests are performed at constant strains, which means

$$\varepsilon_{kl}(t) = \begin{cases} 0 & t < 0 \\ \varepsilon_{kl}^{\mathrm{cst}} & t \geqslant 0 \end{cases} \tag{2}$$

where "cst" means constant values that do not vary with time but may change with position.

Eq. (2) implies: $\lim_{t\to-\infty}\varepsilon_{kl}(t) = 0$.

Then applying the integral by parts to Eq. (1), we can obtain

$$\sigma_{ij}(t) = \left(C_{ijkl}(0) + \int_{0}^{t} \frac{\partial C_{ijkl}(t-\tau)}{\partial(t-\tau)}\mathrm{d}\tau\right)\varepsilon_{kl}^{\mathrm{cst}} = C_{ijkl}(t)\varepsilon_{kl}^{\mathrm{cst}} \tag{3}$$

Eq. (3) implies that the instantaneous stress values are dependent on the instantaneous values of stress relaxation coefficients instead of history effects when the linear viscoelastic materials are subjected to constant strain loading.

3. Micromechanics formulations for effective stress relaxation stiffness

Consider a multiphase viscoelastic composite that is an assembly of many periodic unit cells (UCs). The microstructure of the multiphase composite is illustrated in Fig. 1. Two coordinate systems including $\boldsymbol{x} = (x_1, x_2, x_3)$ and $\boldsymbol{y} = (y_1, y_2, y_3)$ are employed to facilitate the micromechanics formulations. We use x_i as the global coordinates to describe the macroscopic structure and y_i parallel to x_i as the local coordinates to describe the UC (Here and throughout the paper, Latin indices assume 1–3 and repeated indices are summed over their range except where explicitly indicated). We choose the origin of the local coordinate system y_i to be the geometric center of UC.

3.1. Effective stress relaxation stiffness of linear viscoelastic composites

The Eq. (3) may be derived from the following transient potential energy density functional,

$$U(t) = \frac{1}{2}C_{ijkl}(t)\varepsilon_{ij}^{\mathrm{cst}}\varepsilon_{kl}^{\mathrm{cst}} \tag{4}$$

such that

$$\sigma_{ij}(t) = \frac{\partial U(t)}{\partial \varepsilon_{ij}^{\mathrm{cst}}} \tag{5}$$

The effective stress relaxation stiffness of the linear viscoelastic composites can be defined in the following ways,

$$\bar{\sigma}_{ij}(t) = C_{ijkl}^{*}(t)\bar{\varepsilon}_{kl}^{\mathrm{cst}} \tag{6}$$