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The role of constitutive relation in the stability of hyper-elastic spherical membranes subjected to dynamic inflation

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ABSTRACT

In this work the mechanical response of hyper-elastic spherical membranes subjected to dynamic inflation is revisited. Specifically, a comprehensive analysis on the role that the constitutive behaviour of the material has on the mechanical stability of the membrane has been developed. Six different strain-energy functions, frequently used to approximate the constitutive behaviour of elastomeric solids, have been considered: three of the Mooney–Rivlin class and three of the Ogden class. For all the constitutive models used, the material parameters have been obtained from Bucchi and Hearn (2013a, 2013b), where the same set of experimental results was used to calibrate the models. We show that essential features of the dynamic response of the spherical shell are closely related to the strain-energy function selected to describe the constitutive behaviour of the membrane. As reported by Bucchi and Hearn (2013a, 2013b), this issue is frequently overlooked within the literature since too often only one strain-energy function is used to address this type of dynamic problems.

1. Introduction

Since the pioneering works of Mooney (1940), Treolar (1944, 1949) and Rivlin (1948, 1996), the mechanics of hyper-elastic membranes under finite deformations has been a matter of great interest for the continuum mechanics community. The contributions to this topic over the last 70 years are by far too numerous to be cited individually. We focus here the attention on the (approximately) last decade, in which significant advances have been made on the investigation of the mechanical stability of inflated elastomeric shells. As described by Tamadapu and DasGupta (2013) and Kumar and DasGupta (2013), hyper-elastic inflated structures are used in modern applications such as in balloons, self-deploying structures, terrestrial and space structures, airbags and suspensions for cushioning and absorbing shocks. Moreover, we find an important application within the framework of biomedical engineering since the inflation of spherical and cylindrical hyper-elastic shells is frequently taken as a canonical problem to investigate the formation and growth of saccular and fusiform aneurysms (Alhayani, Rodríguez, & Merodio, 2014; David & Humphrey, 2003; Freitas, 2009; Haslach & Humphrey, 2004; Rodríguez & Merodio, 2011; Shah & Humphrey, 1999; Volokh & Vorp, 2008). In this regard, one should pay special attention to the papers of Fu and co-workers (Fu, Pearce, & Liu, 2008, 2012; Fu & Xie, 2010, 2014; Il'ichev & Fu, 2012, 2014; Pearce & Fu,

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2010) who focused on the analysis of bifurcated (bulged) configurations from pressurised tubular and spherical membranes, and explored the stability of the bulging motion. The authors stated that this bifurcation interpretation of the initial formation of aneurysms could provide a theoretical framework under which different mechanisms leading to aneurysms development can be assessed in a systematic manner (Fu & Xie, 2012).

Driven by the applications described above, several authors have attempted to shed light into the key role played by the constitutive behaviour of the membrane material on the mechanical response of the shell. One has to mention the work of Ogden, Saccomandi, and Sgura (2004) who carried out a systematic study on the procedure of *fitting of experimental data* which is regularly used to determine the material parameters of different hyper-elastic constitutive laws. Ogden et al. (2004) pointed out that the fitting process often leads to non-unique optimal parameters. The authors noted that, for a given constitutive model, to use different sets of material parameters markedly affects the solution of boundary value problems, despite these material parameters may be fitted using the same experimental data. With the aim of deepen into the interplay between the material parameters and the mechanical response of elastomeric membranes, Biscari and Omati (2010) studied the inflation of thin spherical shells modelled as a (generalised) Knowles' material (Knowles, 1977). By allowing the constitutive parameters to vary within a wide range of values, Biscari and Omati (2010) showed the close relation between the values assigned to the material parameters and the mechanical stability of the shell. Within the same framework, we highlight the very recent work of Mangan and Destrade (2015) who, as previously did Biscari and Omati (2010), revisited the inflation of spherical shells. Mangan and Destrade (2015) used the 3-parameter Mooney (Mooney, 1940) and the Gent-Gent (Pucci & Saccomandi, 2002) models to describe the membrane behaviour. The authors pointed out the constitutive sensitivity of the problem at hand and the great influence exerted by the material parameters on the stability/instability of the shell response.

Nevertheless, the work of Bucchi and Hearn (2013a, 2013b) seems to be the only research paper which relies on a significant number of different strain energy functions to provide detailed analysis on the interplay between the material constitutive model and the mechanical response of the membrane. It is pointed out by Bucchi and Hearn (2013b) that, in the recent literature, too often only one strain-energy function is used to describe the material behaviour. Even if two or more strain-energy functions are utilised, their parameters are frequently taken from different sources. This practice is a drawback as soon as different sources implies that different sets of experimental data have been used for determining the model parameters. Comparison of alternative analyses based on using different strain-energy functions requires that the parameters of these constitutive models have been obtained using a unique set of experiments and a common identification procedure.

With this in mind, we develop here an analytical study with the aim of exploring specifically the role played by the strain-energy function in the inflation of spherical shells. Unlike the work of Bucchi and Hearn (2013a, 2013b), the constitutive sensitivity analysis developed in the present paper lies within the dynamic regime. While the dynamic inflation of elastomeric shells is less explored than the static one, it was shown by Verron, Khayat, Derdouri, and Peseux (1999), Verron, Marckmann, and Peseux (2001) and Yuan, Zhang, Ren, and Zhu (2010) that inertia has a significant influence on the mechanical stability of spherical balloons. Within this framework, we address in this paper a key issue: different constitutive models calibrated using the same set of experimental data and the same fitting procedure provide different predictions of the dynamic response of the spherical balloon. This highlights how crucial may be the selection of the (appropriate) strain-energy function for (reliable) mathematical modelling of hyper-elastic membranes subjected to dynamic solicitations. As such, our investigation complements recent papers developed under similar premises but within the static regime, see (Biscari & Omati, 2010; Bucchi & Hearn, 2013a, 2013b; Mangan & Destrade, 2015).

The paper is organised as follows. In Section 2 the equation which governs the dynamic inflation of the spherical membrane is deduced. Then, the two loading cases to be analysed are presented: a spherical membrane subjected to (1) constant inflation acceleration and (2) constant inflation pressure step. Section 3 shows the six different strain-energy functions, from which the constitutive relation is derived, that are selected to describe the membrane behaviour. Three constitutive models belong to the Mooney–Rivlin class (Rivlin, 1948) and the other three to the Ogden class (Ogden, 1972). These are *classical* models that have been frequently used in the literature to describe the mechanical behaviour of elastomeric solids. In Section 4 results are presented and analysed combining, systematically, the two loading scenarios and the six constitutive equations. We point out that essential features of the dynamic response of the shell are closely related to the strain-energy function selected to describe the membrane behaviour. Specifically, we focus the attention on the effect that material constitutive modelling has on the mechanical stability of the membrane. This issue is further discussed in Section 5 where a critical overview of the main outcomes of this research is developed. Section 6 summarises the key conclusions obtained from this investigation.

2. Problem formulation

2.1. Governing equation

Consider a spherical membrane of non-linear elastic, isotropic, and incompressible material of density ρ subjected to dynamic inflation. The membrane deformation is described in spherical coordinates (R, Θ, Φ) in the undeformed configuration, and (r, θ, ϕ) in the deformed configuration. Let (R_0, H_0) and (r_0, h_0) denote the membrane midsurface radius and