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A model of a breathing crack with relaxation damping

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ABSTRACT

Relaxation damping is a phenomenon described recently for contact of two purely elastic bodies with infinite coefficient of friction. If a two-dimensional elastic body containing a mixed-mode crack with no sliding between the crack faces is subjected to superimposed oscillations in the normal and tangential directions, then a specific damping appears that is independent of dissipation in the elastic material. It is shown that the rate of energy dissipation due to relaxation damping is proportional to the square of the crack length and depends on the ratio of the normal local prestress to the amplitude of the normal local stress oscillations as well as on the phase shift between both oscillations. In the case of low frequency tangential loading with superimposed high frequency normal oscillations, the system acts as a tunable linear damper. Generalization of the model for the three-dimensional case is discussed.

1. Introduction

A breathing crack model has been considered in a number of studies (Argatov & Sevostianov, 2010; Farrar & Worden, 2007) related to the structural health monitoring (Cheng, Wu, Wallace, & Swamidas, 1999; Matveev & Bovsunovskii, 1999) under the assumption that the opened crack changes the stiffness of a vibrating member containing the small crack. On the other hand, there have been developed quasi-static models (Bui & Oueslati, 2005; Lawn & Marshall, 1998) for cracks with friction. The energy dissipated at the contact interface through frictional micro-slip was studied in a number of papers (Argatov & Butcher, 2011; Davies, Barber, & Hills, 2012; Paggi, Pohrt, & Popov, 2014; Putignano, Ciavarella, & Barber, 2011;Segalman, 2005) as well. The frictional dissipation in a cracked elastic body was studied in Jang and Barber (2011) by Kachanov's simplified model of microcrack interaction (Kachanov, 1987) under the assumption of predominantly compressive periodic loading, so that the small breathing cracks can experience periods of closure and slip.

In the present study, we investigate the problem of quasi-static vibrations of a two-dimensional elastic body containing a breathing crack with infinite friction, which prevents relative sliding the crack faces. As it was first observed in Popov, Popov, and Pohrt (2014) in the case of contact between two purely elastic bodies with infinite coefficient of friction, one can expect that, when the elastic body oscillates under both tensile and shear loading, such a breathing crack will cause purely elastic energy loss associated with the abrupt crack shear opening. This phenomenon is referred to as relaxation damping (Popov et al., 2014) and was studied by the method of dimensionality reduction (Popov & Heß, 2015) first for elastic contacts oscillating in both normal and tangential directions.

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2. A mixed-mode breathing crack

In this section, we develop the leading-order two-dimensional asymptotic model for a small breathing crack under the following simplifying assumptions. The crack is plane and does not extend under quasi-static variable external loading. The crack is isolated and located inside a linearly elastic body. The crack size is relatively small with respect to the characteristic size of the body.

Consider a square elastic plate Ω referred to the Cartesian coordinate system Ox_1x_2 (see Fig. 1). Let a crack of length l be centered at the origin (point O) and located at a distance $L/2$ from the boundary of the elastic plate Ω. The length L is taken to be the characteristic size of Ω, and it is assumed that $l \ll L$.

Let the elastic body Ω be loaded by some normal and shear surface tractions, while body forces are absent. The inertia effects will be neglected as well. We denote by $\sigma_{11}^0, \sigma_{12}^0$, and σ_{22}^0 the stresses at the point O in the elastic body Ω without the crack.

The concept of a small breathing crack implies that the crack is open, when $\sigma_{22}^0 > 0$, and is closed when $\sigma_{22}^0 \leqslant 0$. The potential energy change, $\Delta\Pi$, due to appearance of the small opened crack at the point O is given by the following Griffith's equation (Sih & Liebowitz, 1975):

$$\Delta\Pi = -\frac{\pi l^2}{32G}(\kappa+1)\Big((\sigma_{22}^0)^2 + (\sigma_{12}^0)^2\Big). \tag{1}$$

Here, G is the shear elastic modulus, κ is Kolosov's constant, which can be expressed in terms of Poisson's ratio ν as $\kappa = (3-\nu)/(1+\nu)$ for the plain stress state, and $\kappa = 3 - 4\nu$ for the plain strain state.

Observe that the tensile stress σ_{11}^0, which is acting along the crack, does not enter formula (1). So, for the sake of simplicity, let us assume that $\sigma_{11}^0 = 0$, and, correspondingly, the tensile stress σ_{22}^0 will be related to the tensile strain ε_{22}^0 by the equation

$$\sigma_{22}^0 = \frac{8G}{\kappa+1}\varepsilon_{22}^0.$$

Hence, the control of the crack state can be performed via the control of the normal strain ε_{22}^0, so that in the open-crack state, we have

$$\varepsilon_{22}^0 > 0 \quad \Rightarrow \quad \Delta\Pi < 0, \tag{2}$$

where the energy change $\Delta\Pi$ is given by Griffith's formula (1).

Now, we introduce an additional assumption of an infinite coefficient of friction between the crack faces. In such a case, in the closed-crack state, we will have

$$\varepsilon_{22}^0 \leqslant 0 \quad \Rightarrow \quad \Delta\Pi = 0. \tag{3}$$

We note that the condition $\Delta\Pi = 0$ means that the presence of the mode II crack in the closed-crack state does not change the potential energy of the elastic body Ω, because the infinite friction prevents slip between the crack surfaces.

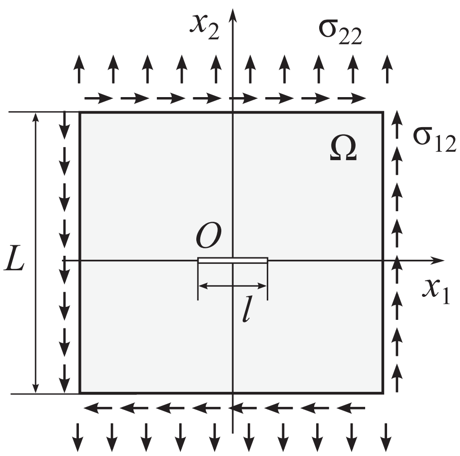

Fig. 1. Configuration of an elastic plate with s small crack under mixed loading.