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The anti-plane shear elastostatic fields near the wedge vertex of an incompressible hyperelastic bimaterial

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ABSTRACT

The anti-plane shear transformation corresponding to a bimaterial wedge problem is studied within the framework of finite elasticity. Two wedges of arbitrary angles are assumed to be incompressible hyperelastic Neo–Hookean materials, and have a common perfect interface and a traction free surface. The resolution of the boundary value problem near the vertex of a bimaterial wedge by means of asymptotic procedure leads to an eigenequation connecting wedges angles and materials properties. Results proved that, contrary to linear elasticity predictions, it is needed to extend development to higher orders because of their contribution in stress singularity.

1. Introduction

It is well known in several engineering applications that the junction between rubber and other materials with different rigidities may improve the structure stiffness. However, this coupling can lead to a stress concentration due to the type of used materials and geometric discontinuities, which can affect the product strength.

From an historically point of view, the theory of continuum mechanics is used to model such a phenomenon with singular elastostatic fields: transformation and stress. Typically, the anti-plane transformation is the most widely studied due to its simplicity and its out-of-plane character. Accordingly, this work is concerned with the investigation and the analysis of elastostatic fields corresponding to the anti-plane transformation of a wedge vertex in incompressible Neo–Hookean hyperelastic bimaterial.

In linear elasticity, the boundary value problem of the two-dimensional multi-material wedges or junctions, associated to anti-plane transformations, is governed by a harmonic equation with Neumann and/or Dirichlet boundary conditions. Its singular nature is well established (Grisvard, 1992) and was analyzed by three methods: asymptotic development, complex variables and transform methods. It was shown that the general solution of the linear boundary value problem corresponding to out-of-plane transformation is an asymptotic development composed of a linear combination of a power and logarithmic types of singularities. The unknowns of this asymptotic development are referred as eigenvalues (exponent orders) and eigenfunctions. The resolution of the boundary value problem by means of asymptotic development leads to an eigenequation having real eigenvalues. Consequently, no oscillatory singularity in elastostatic fields can occur contrary to in-plane deformation case (see reviews papers of Sinclair, 2004a; 2004b and Paggi & Carpinteri, 2008 and the associated references).

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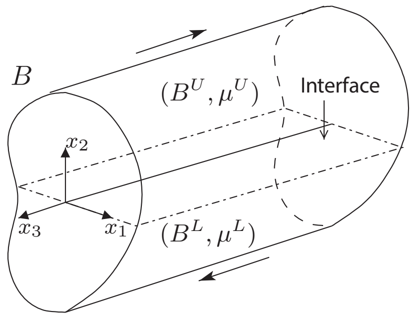

Fig. 1. Three-dimensional bimaterial body.

For elastoplastic constitutive laws at small deformations, the pioneer works of Rice (1966, 1967) for crack and wedge showed that the asymptotic development is made by a power-type singularity. Later, many papers was dedicated to solve the multi-material wedges and junctions problems, including the crack, the interface-crack and the homogeneous wedge. See Zappalorto and Lazzarin (2010) and Loghin et al. (2000) for more details and references. The Linear Elastic Fracture Mechanics (LEFM) and the Elasto-Plastic Fracture Mechanics (EPFM) approaches described below played a prominent role in the investigation and comprehension of crack, defect and singular problems. However, these approaches are based on the kinematic assumption of small deformations which is in contradiction with the unbounded strain field deduced.

Within the framework of finite deformation (Ogden, 1997), only few works focused, in the past five decades, on the analysis of strain and stress fields around cracks, notches, defects, etc. This is due to the formidable complexity of the mathematical problem (Ogden, 1997) which makes the boundary-value problem equations highly nonlinear and very difficult to solve analytically or even numerically. A first analysis of an infinite Neo–Hookean sheet containing a finite crack was carried out by Wong and Shield (1969). Then, in the early 1970s, Knowles and Sternberg (1973, 1974) analyzed the asymptotic deformation field near the tip of a Mode-I plane strain crack for generalized Blatz-Ko compressible hyperelastic solids. Their analysis of the crack problem within the framework of nonlinear elasticity is considered as a fundamental work. Later, Knowles (1977b) performed a mode III local crack analysis for generalized Neo–Hookean incompressible hyperelastic material. This class of hyperelastic potential, depending only on the deformation first invariant, is capable to sustain non-homogeneous anti-plane shear transformation (Knowles, 1976). Some necessary and sufficient mathematical conditions, restricted the hyperelastic potential form, are given by Adkins et al. (1954), Green and Adkins (1960), Knowles (1976), Knowles (1977a) for incompressible and compressible materials to admit non-trivial states of anti-plane shear. Further contributions and applications on the topic are done in Horgan (1995) (see review and references until 1995), (Horgan & Saccomandi, 2001; Hui & Long, 2011). Recently, the question of the compatibility for an overdetermined system of partial differential equations, issued from motion equations of a hyperelastic body undergoing anti-plane shear transformation, has known a resurgence of interest (De Pascalis et al., 2009; Horgan & Saccomandi, 2003; Pucci & Saccomandi, 2013a). To this end, Pucci et al. (2015), Pucci and Saccomandi (2013b) generalized the studies of Knowles (1976), Knowles (1977a) and gave a systematic method to resolve this compatibility problem. To our knowledge, the first analysis of the interface crack of hyperelastic bimaterial junction is studied by Knowles and Sternberg (1983). In two other fundamental papers of Hermann (1989), Hermann (1992), it was shown the existence of multiple second order asymptotic forms for elastostatic fields corresponding to compressible generalized Blatz-Ko hyperelastic bimaterial interface crack. Other interesting works on the topic, using analytical asymptotic development, numerical or experimental investigations was done by Knauss (1970), Ravichandran and Knauss (1989), Gao and Shi (1994), Geubelle and Knauss (1994a, 1994b), Geubelle and Knauss (1995), Geubelle (1995), Ru (1997c), Ru (2002), Krishnan and Hui (2009), Lengyel et al. (2014). The more complicated multi-material wedges or junctions was studied by Liu and Gao (1995), Tarantino (1997), Tarantino (1998) and Arfaoui et al. (2008) for homogeneous wedge and Ru (1997a) for a bimaterial wedge. All previous contributions done with different hyperelastic potentials under the plane deformation or stress hypothesis have shown that there is not oscillatory singularity behavior (except the work of Krishnan & Hui (2009)). An interesting overview in this topic is done by Gao et al. (2008)

In this paper, an asymptotic study of the anti-plane shear problem in an incompressible bimaterial wedge is done. Firstly, the general problem is formulated. Since its resolution is established through an asymptotic procedure, the asymptotic form of displacements is then given. After that, an analytical analysis is carried out and treats separately homogeneous and bimaterial configurations. For the first case, the analytical solution is determined, while for the second case, many geometrical configurations are discussed and a numerical solution is given for each one.

2. Formulation of the anti-plane shear problem

Consider a deformable body composed by two perfectly bonded isotropic incompressible hyperelastic Neo–Hookean materials: an upper one and a lower one. Their shear modulus are μ^U and μ^L, respectively. In the undeformed configuration, this body occupies an infinite cylindrical region B composed of an upper and a lower regions B^U and B^L. The rectangular Cartesian