International Journal of Engineering Science 97 (2015) 40–59

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Static bending and free vibration of a functionally graded piezoelectric microplate based on the modified couple-stress theory

Y.S. Li [a], E. Pan [b,*]

[a] College of Civil Engineering, Hebei University of Engineering, Handan 056038, PR China
[b] Department of Civil Engineering, The University of Akron, OH 44325-3905, USA

ARTICLE INFO

Article history:
Received 24 June 2015
Revised 30 July 2015
Accepted 18 August 2015
Available online 15 September 2015

Keywords:
Modified couple-stress theory
Sinusoidal plate theory (SPT)
Size effect
Functionally graded materials (FGMs)
Piezoelectric plate

ABSTRACT

A size-dependent functionally graded piezoelectric microplate model is developed in this paper. It is based on the modified couple-stress and sinusoidal plate theories. The main advantages of the modified couple-stress theory over the classical couple-stress theory are the introduction of the symmetric couple-stress tensor and the involvement of only one material length-scale parameter. The material properties of functionally graded piezoelectric plate are assumed to vary through the thickness according to a power law. Numerical examples are presented for both static bending and free vibration problems of a simply supported piezoelectric functionally graded microplate. The effects of power-law index of material gradient, material length-scale parameter, plate aspect ratio, and mechanical and electric loadings on the displacement (deflection), electric potential, stress, electric displacement and natural frequency are demonstrated.

1. Introduction

Size-dependent features have been well-known in microstructures involving micro- and nano-scale components and devices (Lam, Yang, Chong, Wang, & Tong, 2003; McFarland & Colton, 2005). Since the classical elasticity theory is not capable of predicting such size effects due to the lack of a material length-scale parameter, higher-order continuum theories need to be applied. The couple-stress theory is one of the higher-order continuum theories, which contains two material length-scale parameters as elaborated by Mindlin and Tiersten (1962), Toupin (1962), and Koiter (1964). Recently, a modified couple-stress theory, which contains only one material length-scale parameter, was proposed by Yang, Chong, Lam, and Tong (2002). Based on the modified couple-stress theory, several size-dependent structural models have been proposed to understand the size effect in small-scale structures. For example, static and dynamic problems of beams were recently solved using the modified couple-stress theory (Akgoz & Civalek, 2011; Asghari, Rahaeifard, Kahrobaiyan, & Ahmadian, 2011; Chen, Li, & Xu, 2011; Dehrouyeh-Semnani & Nikkhah-Bahrami, 2015; Liu & Reddy, 2011; Ma, Gao, & Reddy, 2008, 2010; Mohammad-Abadi & Daneshmehr, 2015; Park & Gao, 2006; Reddy, 2011; Simsek & Reddy, 2013).

As far as the plate is concerned, Tsiatas (2009) first developed a Kirchhoff plate model for static analysis of microplates by using the modified couple-stress theory. Jomehzadeh, Noori, and Saidi (2011) used this model to study the vibration of microplates. Asghari (2012) dealt with the Kirchhoff plate theory of isotropic plates including the geometric nonlinearity.

* Corresponding author. Tel.: +1 330-972-6739.
E-mail address: pan2@uakron.edu (E. Pan).

http://dx.doi.org/10.1016/j.ijengsci.2015.08.009
0020-7225/

Nomenclature

a,b	length, width of the plate
c_{ijkl} or c_{ij}	elastic coefficients
e_{ijk} or e_{ij}	piezoelectric coefficients
$f(z)$	shape function for the transverse shear strain
$g(z)$	the derivative of $f(z)$
h	thickness of the plate
l	material length-scale parameter
m_{ij}	couple-stress tensor
n_x, n_y	direction cosines of the unit normal to the boundary
q	z-direction loading applied on the upper surface of the plate
u_i	displacements along x,y,z-directions
u,v,w	displacements in x-,y-,z-directions on the middle plane
x,y,z	coordinate variables in x-,y-,z-directions
D_i	electric displacements
E_i	electric fields
G	shear module
$I_0, I_1, J_1, I_2, J_2, K_2$	mass inertias
M, N	numbers of the Fourier series
N_{ij}, M_{ij}, P_{ij}, Q_{ij} , R_{ij}, S_{ij}, T_{ij}, K_{ij}	stress and couple-stress resultants
V_0	external electric voltage
V_u ,V_l	volume fraction at the upper or lower materials
σ_{ij}	stress tensor
ε_{ij} or γ_{ij}	strain tensor
χ_{ij}	curvature tensor
θ_i	rotations
λ	gradient index
φ	electric potential on the middle plane
ω	circular frequency
ψ_x , ψ_y	rotations of the middle plane in x- and y-directions
μ_{ik}	dielectric coefficients
ρ	mass density
η	normalized electro-mechanics loading ratio parameter
δK	virtual kinetic energy
δW	virtual work done by external forces
δU	virtual strain energy
Y, Y_u, Y_l	material property, material properties on the upper, lower surfaces
Φ	electric potential

Ma, Gao, and Reddy (2011) and Ke, Wang, Yang, and Kitipornchai (2012) proposed a new Mindlin plate model to account for the effects of transverse shear-deformation and rotary inertia in moderately thick microplates. Using the modified couple-stress theory and a meshless method, Roque, Ferreira, and Reddy (2013) derived the static bending solution of simply supported and clamped, isotropic microplates according to the first-order shear-deformation plate theory. Based on the Hamilton's principle, Gao, Huang, and Reddy (2013) proposed a non-classical third-order shear-deformation plate model where the modified couple-stress theory is used. It is shown that the new third-order shear-deformation plate model contains the non-classical Reddy–Levinson beam model and Mindlin couple-stress plate model as special cases. Kim and Reddy (2013) presented analytical solutions of the general third-order plate theory using the Navier solution technique for bending, vibration, and buckling problems. Thai and Thuc (2013) presented a size-dependent model for bending and free vibration of functionally graded material (FGM) plate based on the modified couple-stress and sinusoidal shear-deformation theories (Touratier, 1991). Based on a modified couple-stress theory, a model for sigmoid FGM nanoplates on an elastic medium was proposed by Jung, Han, and Park (2014). Ke, Yang, Kitipornchai, and Wang (2014) investigated the axisymmetric postbuckling of FGM annular microplates based on the modified couple-stress theory, Mindlin plate theory and von Kármán geometric nonlinearity. Shaat, Mahmoud, Gao, and Faheem (2014) proposed a new Kirchhoff plate model using a modified couple-stress theory to study the bending behavior of nano-sized plates, including surface energy and microstructure effects.

Piezoelectric materials have been widely used as sensors and actuators in control systems due to their excellent electro-mechanical properties, design flexibility, and efficiency to convert electrical energy into mechanical energy or vice versa. Traditional piezoelectric sensors and actuators are often made of several layers of different piezoelectric materials. Many theoretical and mathematical models have been presented for laminated composite structures with piezoelectric sensors and actuators