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An Eulerian formulation of nonlinear thermomechanics and electrodynamics of moving anisotropic elastic solids

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ABSTRACT

In this paper, the formulation of nonlinear thermomechanics and electrodynamics of deformable materials proposed by Eringen and Maugin (1990) is restructured using the thermomechanical formulation of Green and Naghdi (1977, 1978). Constitutive equations for thermoelastic solids are typically proposed using Lagrangian forms of deformation variables. Consequently, previous developments of constitutive equations for thermoelastic electrodynamic materials proposed Lagrangian forms of electrodynamic quantities. In contrast, here, an Eulerian formulation is proposed which naturally unifies the constitutive modeling. In particular, the formulation does not use electrodynamic body force or body couple terms and the nonlinear constitutive equations for thermomechanical and electrodynamic variables are determined by derivatives of a single Helmholtz free energy function. Also, the constitutive equations for electrodynamic contributions to the stress tensor, the polarization and magnetization vectors depend naturally on the specific functional form of the Helmholtz free energy and need not be proposed independently.

1. Introduction

Eringen and Maugin (1990) emphasize that the classical developments of the theories of electrodynamics and continuum thermomechanics have been developed independently of each other. With electrodynamics ignoring deformations and continuum thermomechanics ignoring electromagnetic effects. Classical works which combine these two approaches and present solutions to various problems include e.g. Penfield and Haus (1967); Hutter and van de Ven (1978); Eringen and Maugin (1990). Recent efforts to develop electroactive polymers as actuators capable of producing large deformations in response to electric and magnetic fields have stimulated renewed interest in unifying theories of electrodynamics and continuum thermodynamics, especially for static response (e.g. Dorfmann & Ogden, 2005, 2006, 2014). Mention is also made of an alternative approach based on implicit constitutive equations for electroelastic and magnetoelasic bodies (Bustamante & Rajagopal, 2013, 2015).

Typically, the development of electrodynamics uses proposals for specific constitutive equations to model phenomena like electromagnetic, ferromagnetic, piezoelectric and dielectric material response. In contrast, the development of continuum thermodynamics emphasizes the unified nature of nonlinear theories, with response functions determined by derivatives of specific energies. One confusing issue has been proposals of different expressions for the electrodynamic body force and its complementary electrodynamic stress tensor [e.g. Hutter and van de Ven, (1978) discuss five models of the influence of electrodynamics on

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the body force and stress tensor]. However, for classes of materials it has been recognized (e.g. Hutter & van de Ven, 1978; Eringen & Maugin, 1990) that these different formulations are mathematically equivalent. Specific reference is made to the formulation presented in Eringen and Maugin (1990, Eqs. (3.10.13–3.10.15), which proposes balance laws without the need for introducing electrodynamic body force and body couple. Recent works (e.g. Dorfmann & Ogden, 2006, 2014) also emphasize that the influence of the electromagnetic body force can included in a unified manner in the energy potentials which determine the total stress.

Another difference between the theory of electrodynamics and thermoelasticity is that Maxwell's equations are naturally formulated in an Eulerian form, whereas, the constitutive equations of thermoelasticity are usually formulated in a Lagrangian form. Specifically, the constitutive equations of a thermoelastic solid are taken to be functions of the total deformation gradient from an arbitrary, but fixed reference configuration. Since a uniform homogeneous thermoelastic material has a unique shape and density in any unstressed state at reference temperature it is common to take the reference configuration to be unstressed. In contrast, uniform homogeneous elastic-plastic and elastic–viscoplastic materials can have different shapes in unstressed states at reference temperature. If they are porous materials then they can also have different densities in these unstressed states. In this regard, the constitutive equations for elastic–plastic and elastic–viscoplastic materials should be more like those for fluids than those for elastic solids, since the arbitrary choice of the reference configuration should not influence the response of these materials (Rubin, 1996, 2001, 2012).

Eckart (1948) seems to be the first to propose that the constitutive response of elastic-plastic materials should be formulated in terms of an elastic deformation tensor which is determined by integrating an evolution equation that includes a term controlling the rate of inelasticity. The evolution equation for this elastic deformation tensor has an Eulerian form. Leonov (1976) proposed a similar evolution equation for an elastic deformation tensor that was used to determine constitutive equations for the elastic–viscoplastic response of polymeric liquids. The constitutive equations proposed by Eckart (1948) and Leonov (1976) were valid for elastically isotropic response. Rubin (1994) generalized this approach for elastically anisotropic elastic–viscoplastic materials by proposing evolution equations for a triad $\mathbf{m}_\mathrm{i}$ ($\mathrm{i} = 1, 2, 3$) of linearly independent microstructural vectors. It has been shown (Rubin, 1996, 2001, 2012) that this formulation removes unphysical arbitrariness in the constitutive equations of choices of the reference and intermediate stress-free configurations in standard formulations of large deformation elastic–plastic response. More recently, Rubin and Nadler (2015) have emphasized that unphysical arbitrariness of the choice of the reference configuration of constitutive equations for anisotropic hyperleastic materials can also be removed by using these microstructural vectors $\mathbf{m}_\mathrm{i}$ instead of the total deformation gradient. This approach leads to an Eulerian formulation of hyperelasticity.

The objective of this paper is to present a simple unified Eulerian formulation of the balance laws and constitutive equations for moving nonlinear anisotropic thermoelastic electrodynamic solids. To this end, the developments in Eringen and Maugin (1990) are combined with the thermomechanical approach presented in Green and Naghdi (1977, 1978, 1984). Specifically, the structure of the conservation of mass, the balances of linear momentum, entropy and angular momentum remain unchanged relative to those of a thermoelastic solid, except that the total Cauchy stress now depends on both thermomechanical and electrodynamic fields. In particular, the balances of linear and angular momentum do not include additional fields like an electrodynamic body force and body couple. In contrast, the unifying nature of the balance of energy includes an electrodynamic energy density and an electrodynamic flux, which couple thermoelastic and electrodynamic effects.

Recently, Weile et al. (2014) have developed a proper formulation of Maxwellian electrodynamics for continuum mechanics. However, they did not discuss the thermomechanical theory, the nature of coupling of mechanical, thermal and electrodynamics in the balance of energy equation, or discuss constitutive equations. Specifically, they developed a universal time formulation of the basic equations of electrodynamics using a four-dimensional metric tensor in space-time and presented the balance laws as differential equations for an unambiguous electromagnetic tensor, which emphasizes "the centrality of relativity theory to the formulation of electrodynamic equations in the vicinity of mechanical motion". With reference to formulations in the literature they stated that "…there is an insistence on formulating the equations in the reference domain, a concept with no meaning in the realm of electromagnetics and possibly no physical meaning to boot". With reference to their universal time formulation they stated that "By insisting on convective rather than reference coordinates, the work presented here has removed the ambiguity of previous formulations by allowing the theory to be associated with quantities that can be measured". The Eulerian formulation presented in this paper seems to be consistent with these statements. In particular, the indices i of the microstructural vectors $\mathbf{m}_\mathrm{i}$ in this Eulerian formulation are considered to be convected with the material. Also, the quantities $\{\mathbf{e}^*, \mathbf{h}^*, \mathbf{j}^*\}$ defined in (4.2) depend on the electric intensity vector $\mathbf{e}$, the magnetic intensity vector $\mathbf{h}$, the current density vector $\mathbf{j}$, the material velocity $\mathbf{v}$, the electric displacement vector $\mathbf{d}$, the magnetic induction vector $\mathbf{b}$, and the free charge e, in a similar manner to the relationships presented in Tables 1, 2 and Eq. (73) in (Weile et al., 2014) associated with their universal time formulation.

An outline of this paper is as follows. Section 2 presents some basic kinematical and tensor results and Section 3 describes an Eulerian formulation of deformation measures. Section 4 summaries the balance laws of electrodynamics for a moving body and Section 5 presents the balance laws for coupled thermomechanical electrodynamic theory. Section 6 discusses jump conditions across a singular surface and a line, and Section 7 records transformation relations for invariance under Superposed Rigid Body Motions (SRBM). Section 8 develops constitutive equations for a nonlinear anisotropic thermoelastic electrodynamic solid and Section 9 presents constitutive equations for an isotropic solid. Section 10 presents conclusions. Also, Appendix A records the equivalence of quantities defined in (Eringen and Maugin, 1990) with those used in the present paper and Appendix B records some mathematical details.