International Journal of Engineering Science 97 (2015) 84–94

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Buckling analysis of size-dependent nonlinear beams based on a nonlocal strain gradient theory

Li Li*, Yujin Hu

State Key Laboratory of Digital Manufacturing Equipment and Technology, School of Mechanical Science & Engineering, Huazhong University of Science & Technology, Wuhan, Hubei 430074, China

ARTICLE INFO

Article history:
Received 20 August 2015
Revised 25 August 2015
Accepted 27 August 2015
Available online 2 October 2015

Keywords:
Buckling
Size-dependent nonlinear beams
Nonlocal strain gradient theory
Strain gradient theory
Nonlocal continuum theory

ABSTRACT

A size-dependent nonlinear Euler–Bernoulli beam is considered in the framework of the nonlocal strain gradient theory. The geometric nonlinearity due to the stretching effect of the mid-plane of the size–dependent beam is considered here. The governing equations and boundary conditions are derived by employing the Hamilton principle. The post-buckling deflections and critical buckling forces of simply supported size-dependent beams are analytically derived. The derived results are compared with those of strain gradient theory, nonlocal elasticity theory and classical elasticity theory. It is found that the post-buckling deflections can be increased by increasing the nonlocal parameter or decreasing the material characteristic parameter. The high-order buckling deflections are more sensitive to size-dependent parameters than the low-order buckling deflections. Furthermore, the critical buckling force can be increased by decreasing the nonlocal parameter when the nonlocal parameter is larger than the material characteristic parameter, or increasing the nonlocal parameter when the nonlocal parameter is smaller than the material characteristic parameter.

1. Introduction

With the rapidly changing development of micro-electro-mechanical systems (MEMS) and nano-electro-mechanical systems (NEMS) including nano-optomechanical systems (NOMS), nanosensors, nanoactuators and drug delivery devices, the study for basic structural elements including rods, beams and plates at micro/nano-scaled is always of scientific interest and fundamental significance, and therefore draw unprecedented attention. It has been reported (Falvo et al., 1997; Lourie, Cox, & Wagner, 1998; Waters, Riester, Jouzi, Guduru, & Xu, 2004) that, the possible form of instability for micro/nano-scaled beams is buckling, which means that these micro/nano-scaled beams may take place some sudden sideways failures when subjected to a high compressive stress (note the compressive stress at the buckling point is less than the ultimate compressive stress that the considered material is capable of withstanding). That is, the original equilibrium point of micro/nano-scaled beams can become statically unstable and therefore can lead to a failure mode. When the axis compressing force is beyond the critical value, the original equilibrium will become unstable and the actual motions will take place at a new equilibrium position. Under such case, the post-buckling analysis of beams cannot be answered well by using linear theory and has to be reassessed by considering nonlinear theory.

---

* Corresponding author. Tel.: +86 27 87543972.
E-mail address: lili_em@hust.edu.cn, lili_em@126.com (L. Li).

http://dx.doi.org/10.1016/j.ijengsci.2015.08.013
0020-7225/© 2015 Elsevier Ltd. All rights reserved.

Significant size-dependent effects on the mechanical and physical properties of micro/nano-scaled structural systems have been observed in many experimental investigations. Therefore, the buckling behaviour of elastic materials with micro/nano-structure cannot be described adequately by using the classical linear elasticity theory, which is associated with the concepts of homogeneity and locality of stress. When size-dependent effects on micro/nano-scaled structural systems are important, the state of stress has to be defined in a nonhomogeneous (or nonlocal) manner. Due to the fact that analysing the size-dependent buckling behaviour of micro/nano-scaled structural systems may provide important information for the design procedures or potential experiments of micro/nano-scaled devices, some macroscopic continuum mechanics (including nonlocal elasticity theory, strain gradient theory and nonlocal strain gradient theory) have been successfully developed and employed to assess the size-dependent effect.

In contrast to classical elasticity theory, the stress of nonlocal elasticity theory at a reference point accounts for not only the strain at the reference point, but also the strains at all points in the body (Eringen, 1983). Lots of works have been carried out based on Eringen's nonlocal elasticity theory to study the size-dependent effects on the mechanical behaviour of linear and nonlinear structural systems, see, e.g., (Ali-Asgari, Mirdamadi, & Ghayour, 2013; Dai, Wang, & Wang, 2015; Kuang, He, Chen, & Li, 2009; Polizzotto, 2001; Reddy, 2007; Reddy & Pang 2008), they showed nonlocal elastic models can only produce softening stiffness with increasing the nonlocal parameter. Based on the Eringen's nonlocal model, Reddy (2007) studied the bending, buckling and vibration of different linear beam models. Challamel et al. (2014) presented an analytical method to calibrate the nonlocal parameter for buckling analysis of micro-structures. However, it was reported that the capability of nonlocal elasticity theory provided to study the size-dependent effects on the mechanical properties of small-scaled structures may exist some limited problems (Eltaher, Hamed, Sadoun, & Mansour, 2014; Li, Yao, Chen, & Li, 2015a; Lim, Zhang, & Reddy, 2015; Ma, Gao, & Reddy, 2008). There is an unresolved paradox that bending solutions of nonlocal nanobeams in some cases were found to be identical to the classical local solution, i.e., the small scale effect cannot be observed at all (Challamel & Wang 2008). Thus, by employing the nonlocal elasticity theory, the stiffness enhancement effects observed from experimental studies and as well as the strain gradient theory (Lam, Yang, Chong, Wang, & Tong, 2003) cannot be predicted well. In addition, the nonlocal elastic model needs no extra boundary conditions for buckling analysis and this is also questionable since extra (non-standard) boundary conditions are expected to be added in the higher-order elasticity model (Ma et al., 2008; Polizzotto 2003; Volokh & Hutchinson 2002).

The gradient elasticity theories (Lam et al., 2003; Mindlin, 1964, 1965; Toupin, 1962) assume that the materials must be considered as atoms with higher-order deformation mechanism at micro/nano scale rather than just modelled them as collections of points. The gradient elasticity theories add some additional higher-order strain gradient terms to the classical equations of elasticity. Lam et al. (2003) presented a modified strain gradient elasticity theory by constructing additional equilibrium equations to assess the behaviour of higher-order strain gradients. Based on the modified strain gradient elasticity and the modified couple stress theories, Akgöz and Civalek (2012) studied the buckling analysis of micro-scaled Euler–Bernoulli beams. Many relative studies have been performed to deal with the static and dynamic problems via various gradient theories, they showed a stiffness enhancement effect with increasing the gradient parameters (see, e.g., Akgöz & Civalek 2013; Ansari, Gholami, & Rouhi, 2013a; Kong, Zhou, Nie, & Wang, 2008; Ma et al., 2008; Mohammad-Abadi & Daneshmehr 2014; Şimşek & Reddy 2013; Srinivasa & Reddy 2013 and references therein). The nonlinearity is a common phenomenon due to the mid-plane stretching and the nonlinear problems are very noteworthy since two immovable supports are often used in micro/nano-scaled beams. Xia, Wang and Yin (2010) studied the static bending, post-buckling and free vibration of nonlinear beams via modified couple stress theory. Ramezani (2012) derived the equations of motion of a Timoshenko microbeam with both the size-dependent and geometric nonlinear effects. Ansari, Gholami, Shojaei, Mohammadi and Sahmani (2013b) analysed the buckling problem of a functionally graded microbeam based on a strain gradient theory. Ghayesh, Farokhi and Amabili (2013) studied the nonlinear dynamics of micro-scaled beams via the modified couple stress theory. Sahmani, Bahrami and Ansari (2014) studied the nonlinear free vibration of functionally graded third-order shear deformable microbeams via a modified strain gradient elasticity theory. Şimşek (2014) analysed the nonlinear static and free vibration of microbeams based on the nonlinear elastic foundation via modified couple stress theory. Dai et al. (2015) studied the nonlinear dynamics of cantilevered microbeams via modified couple stress theory. Karparvarfard, Asghari and Vatankhah (2015) derived a geometrically nonlinear beam model based on the second strain gradient theory. Ghayesh and Farokhi (2015) examined the complex dynamics of a micro-scaled nonlinear beam under a time-dependent longitudinal load.

From the discussions above, it is found that the size-dependent models presented in the strain gradient theories and the nonlocal elasticity model are two entirely different models with mechanical and physical characteristics of small-scaled materials and structures. Recently, Lim et al. (2015) developed the nonlocal strain gradient theory to bring both of the length scales into a single theory via the thermodynamics framework. The nonlocal strain gradient theory is focused on assessing the true effect of the two length scales on the mechanical and physical responses of size-dependent structures. More recently, based on the nonlocal strain gradient theory, Li, Hu and Ling (2015b) studied the flexural wave propagation in size-dependent functionally graded beams.

From the literature mentioned above, it is evident that there are strong scientific requirements to analyze the buckling behaviour of size-dependent beams with both the size-dependent and geometric nonlinear effects so that the size-dependent effect on the post-buckling problem of size-dependent beams can be accurately assessed. In this study, a size-dependent nonlinear Euler–Bernoulli beam formulation is developed in the framework of the nonlocal strain gradient theory. By using the beam formulation, the post-buckling deflections and critical buckling forces of simply supported beams are analytically derived. The derived results are compared with those of strain gradient theory, nonlocal elasticity theory and classical elasticity theory.