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A complete frictional contact: The transition from normal load to sliding

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ABSTRACT

The problem of complete frictional contact between a square elastic block and an elastically similar half-plane is considered when the block is first pressed normally into the half-plane and then a monotonically increasing shear force is applied until sliding is achieved. Numerical results show that for moderate levels of shear force (i.e. less than 50% of the sliding condition) very different qualitative behaviour is observed for high and low coefficients of friction. However, for high shear forces (i.e. near to the sliding condition), the response for high and low friction coefficients is similar.

1. Introduction

The Hertzian contact, which exemplifies those in which both contacting bodies are capable of idealisation as half-planes, is well understood. Because this idealisation means the two bodies have equivalent domains, if they are made from the same material, the problem is fully uncoupled, so that, when the normal contact is developed, particles away from the centreline displace laterally by the same amount, and no shearing tractions are developed. Thus normal indentation leads to a fully stuck interface, regardless of the coefficient of friction. If a monotonically increasing shearing force is then applied, slip zones will form at both contact edges, which will migrate inwards, and the last point to slip is at the centre of the contact. This solution has been known for 75 years, and it is associated with the names Cattaneo (1938) and subsequently Mindlin (1949) .

Here, we wish to understand the response of the simple complete contact shown in Fig. 1, which is formed between a square elastic block and an elastically similar half-plane that is subjected to the same quasi-static loading regime described above: namely, a normal load is first applied to form the contact, and then a smoothly increasing shear force is applied until sliding is achieved. Although an elasticity formulation for a rectangular contact with simple boundary conditions (e.g. constant normal displacement and frictionless) is feasible (Khadem & O'Connor, 1969a, 1969b), with the kind of complicated boundary conditions expected here, this does not seem realistic.

This problem has been studied in some detail by Churchman and Hills (2006) and later by Karuppanan and Hills (2008a). The approach adopted by these authors was to use a combination of the finite element method and an asymptotic analysis of the contact edge. The motivation for using asymptotic solutions is that the contact edges almost always sustain a singular state of stress, which is exceptionally difficult to capture accurately using a purely numerical approach. The corner asymptotes are semi-infinite in nature, and the only physical dimension that enters their formulation is the contact-edge

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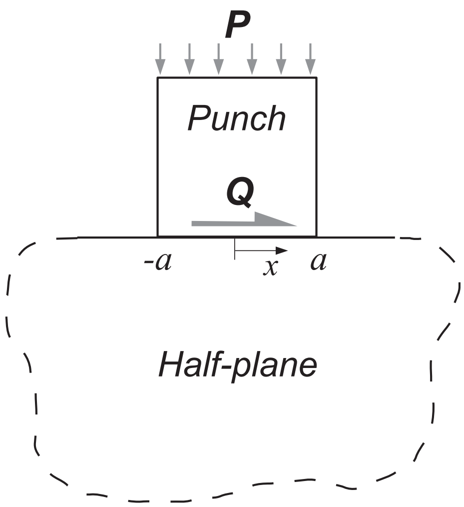

Fig. 1. Complete contact between a square punch of width 2a and an elastically similar half-plane subject to a normal load, P, and a shear load, Q.

angle. This means that the results obtained are universal in the sense that they apply to any contact of the same edge angle, but they cannot be used to describe the stress state at the interior of the contact, where the finite geometry of the problem is important.

Which asymptotic solution is appropriate depends on the interface conditions arising at the contact edges. When the contact is fully adhered, the contact edge can be modelled as a notch, so the series expansion solution developed by Williams (1952) is applicable. Churchman and Hills (2006) investigated the implications of this solution for the contact shown in Fig. 1, and derived the behaviour arising on the initial application of the normal load, P. Making use of a result due to Hills and Dini (2004), Churchman and Hills (2006) demonstrated that the contact remains fully adhered on the application of the normal load if the coefficient of friction, f, is greater than 0.543 (for this particular contact angle); otherwise both contact edges slip outwards. Finite element results obtained by Churchman and Hills (2006) revealed that, when $f > 0.543$, slip does not initiate until a shear of approximately 55% that required to cause full sliding is applied, and that slip initiates from the interior of the contact in these cases.

When slip initiates from the interior of the contact, corner asymptotes provide no information on their evolution, and a purely numerical approach must be used. On the other hand, when slip initiates from the contact edge, the asymptotic solution for a wedge sliding on a half-plane developed by Gdoutos and Theocaris (1975) and Comninou (1976) can be employed. Karuppanan and Hills (2008b) investigated the implications of the sliding asymptote for complete contacts, and elucidated the numerous contact-edge states that may occur depending on the contact angle and the coefficient of friction. Karuppanan, Churchman, Hills, and Giner (2008) also presented more detailed results on contact-edge behaviour for a square block sliding on a half-plane. The results for this contact angle revealed that the tractions at the leading edge (i.e. the right edge in Fig. 1) are power-order singular, but that three types of contact-edge behaviour are possible at the trailing edge, depending on the coefficient of friction. These are:

1. If $f < 1/\pi$, the tractions are power-order singular, but note that the order of singularity is weaker than at the leading edge.
2. If $1/\pi < f < 0.392$, the trailing edge experiences a power-order bounded response but is in intimate contact.
3. If $0.392 < f$, the trailing edge separates, and the contact pressure is locally square-root bounded.

Thus, Williams' solution (Williams, 1952) describes contact-edge behaviour on the initial application of the normal load, and up to moderate values of shear load if the coefficient of friction is sufficiently large (i.e. if $f > 0.543$). Conversely, the behaviour at high shear loads near the sliding condition is accurately described by the formulation due to Gdoutos and Theocaris (1975) and Comninou (1976). The transition between these two states, however, must be tackled numerically. Churchman and Hills (2006) investigated this transition problem by plotting out the qualitative response to be expected on a plot of the load ratio (i.e. Q/fP) vs. the coefficient of friction. Additional refinements to this plot were later presented by Karuppanan and Hills (2008a), and these authors also examined two other contact angles in this way.

Our intention is to examine in detail the evolution of contact behaviour during the transition from full stick to sliding by focusing on two particular example problems: (i) a high-friction case when $f = 0.8$ and (ii) a low-friction case when $f = 0.3$. These examples values have been chosen because the asymptotic analysis described above suggests that qualitatively different behaviour should be expected for coefficients of friction above and below 0.543. But aside from this consideration, the precise values chosen are somewhat arbitrary.