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Scattering of acoustic waves on a planar screen of arbitrary shape: Direct and inverse problems

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A R T I C L E I N F O

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A B S T R A C T

Scattering of plane monochromatic acoustic waves on a planar screen of arbitrary shape is considered (direct problem). The 2D-integral equation for the pressure jump on the screen is discretized by Gaussian approximating functions. For such functions, the elements of the matrix of the discretized problem take the form of a standard one-dimensional integral that can be tabulated. For regular grids of approximating nodes, the matrix of the discretized problem has the Toeplitz structure, and the corresponding matrix–vector products can be calculated by the Fast Fourier Transform technique. The latter strongly accelerates the process of iterative solution. Examples for an elliptic screen subjected to incident fields with various wave vectors are presented. The problem of reconstruction of the screen shape from the experimentally measured amplitude of the far field scattered on the screen (inverse problem) is discussed. Screens which boundaries are defined by a finite number of scalar parameters are considered. Solution of the inverse problem is reduced to minimization of functions that characterize deviation of experimental and theoretical amplitudes of the far field scattered on a screen. Local and global minima of these functions with respect to the screen shape parameters are analyzed. Optimal frequencies for efficient solution of the inverse problem are identified.

1. Introduction

The problem of acoustic wave scattering on screens has important applications in hydro-acoustics. This problem is first reduced to a 2D-integral equation for the pressure jump on the screen surface (see, e.g., Colton & Kress, 1987), and the boundary element method is used for its numerical solution (Shaw, 1979). In this method, the integral equation is discretized by division of the screen surface into a set of small subregions (boundary elements), and inside each element, the solution is approximated by standard functions (e.g., polynomial splines) with unknown coefficients. Using the method of moments or the collocation method the problem is reduced to a finite system of linear algebraic equations for these coefficients (discretized problem). Its matrix is non-sparse, and the matrix terms are integrals over the boundary elements. For high frequency of the incident field, this matrix is large, and only iterative methods are efficient. As a result, time-consuming operation of the matrix–vector product should be performed at every step of the iteration process.

An efficient method of solving the integral equations of the scattering problems was proposed in Kanaun and Levin (2013) and Kanaun (2014). In this method, discretization of the integral equations is carried out using a set of identical approximating functions centered at the nodes of a regular grid. As the result, the matrix of the discretized problem has Toeplitz's

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properties, and the corresponding matrix–vector products can be calculated by the Fast Fourier Transform (FFT) technique. The latter accelerates substantially the process of the iterative solution of the discretized problems. In the work of Kanaun (2014), Gaussian approximating functions were used for discretization of the integral equation of elastic wave scattering on a planar crack. The theory of approximation by Gaussian functions was developed by Maz'ya and Schmidt (2007). In the present work, this method is applied to the problem of acoustic wave scattering on a planar screen of arbitrary shape. This allows substantial reduction in time and memory requirements in comparison with conventional numerical methods. The inverse scattering problem – determination of the screen shape from the data of the far scattered field – can be also successfully solved by the method.

In the inverse problem, attention is focused on properties of the functionals that characterize deviation of the predicted and experimental amplitude of the far field scattered on the screen (see, e.g., Colton & Kress, 1998). Screens which boundaries are defined by a finite number of scalar parameters are considered. The corresponding functionals become functions of these parameters, and the problem is reduced to seeking minimum of these functions. It is shown that these functions can have a number of local minima. As a result, numerical methods of seeking the minimum can converge to a local but not global minimum depending on the initial guess. The number and positions of the local minima depend on frequency of the incident field. It is shown that the inverse problem can be solved successfully if the wave number α of the incident field satisfies the condition $\alpha L = O(50)$, where L is the characteristic size of the screen. In this case, the number of minima is reduced to one global minimum.

The structure of the paper is as follows. In Section 2, the integral equation of the scattering problem for screens is discussed. In Section 3, approximation by Gaussian functions is considered, and in Section 4, the integral equation of the scattering problem is discretized by Gaussian approximating functions. In Section 5, the far scattered field, differential and total cross-sections of planar screens are considered. Examples of the numerical solutions of the scattering problems for an elliptic screen are presented in Section 6. The inverse problem is considered in Section 7.

2. Integral equations of the scattering problem for a screen

Consider an infinite liquid medium that contains a planar screen with surface Ω bounded by the contour Γ (Fig. 1). Let a plane monochromatic incident pressure wave $P^0(x,t)$

$$P^0(x,t) = p^0(x)e^{i\omega t}, \quad p^0(x) = ae^{-i\alpha(\mathbf{n}^0\cdot\mathbf{x})} \tag{1}$$

propagate in the medium and be scattered on the screen. Here ω is frequency, t is time, $\alpha = \omega/c$ is the wave number of the incident field, c is the wave velocity; $\mathbf{n}^0$ is its wave normal, and $\mathbf{n}^0 \cdot \mathbf{x}$ is the scalar product of the vectors $\mathbf{n}^0$ and $\mathbf{x}$ ($\mathbf{x}$ is the vector of a point x in the 3D-medium). The pressure $P(x,t)$ in the medium with the screen has the form

$$P(x,t) = p(x)e^{i\omega t}, \tag{2}$$

where the amplitude $p(x)$ satisfies Helmgholtz equation (Pierce, 1981)

$$\Delta p + \alpha^2 p = 0, \tag{3}$$

(Δ is the 3D-Laplace operator) and the boundary condition on the screen surface Ω is

$$\left.\frac{\partial p}{\partial n}\right|_\Omega = n_i \left.\frac{\partial p}{\partial x_i}\right|_\Omega = 0, \tag{4}$$

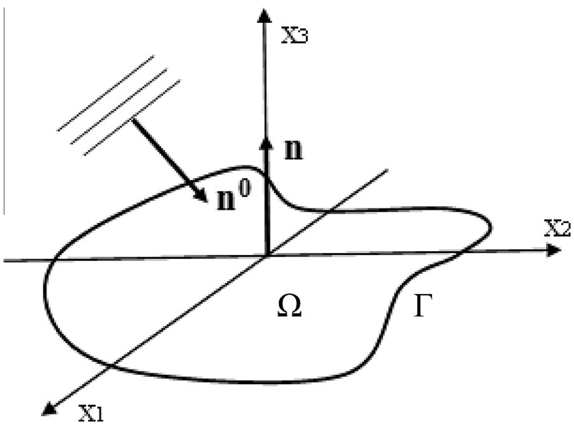

Fig. 1. Planar screen subjected to an incident wave with the wave normal $\mathbf{n}^0$.