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Size-dependent thermal buckling analysis of micro composite laminated beams using modified couple stress theory

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ABSTRACT

The thermal effect on size-dependent buckling analysis of micro composite laminated beams is studied based on the modified couple stress theory. The governing equations and boundary conditions are obtained by using the principle of minimum potential energy. The effect of shear deformation is investigated by considering the Euler–Bernoulli, Timoshenko and Reddy beam theories. In addition, the effect of boundary condition by using the hinged–hinged, clamped–hinged and clamped–clamped boundary conditions and the effect of lamination by applying four kinds of cross ply laminate i.e. [0, 0, 0], [0, 90, 0], [90, 0, 90] and [90, 90, 90] are investigated. Using the Fourier series expansions, the governing equations and boundary conditions are solved analytically just for hinged–hinged boundary condition. Also by using the generalized differential quadrature (GDQ) method, the governing equations and boundary conditions are solved numerically for all of the boundary conditions. Comparison between results obtained by numerical solution, analytical solution and those obtained by literature reveals the accuracy of GDQ method.

1. Introduction

Increasing the use of materials with microstructure in micro-electro-mechanical systems (MEMS) such as microswitches and microrobots has recently motivated many researchers to study the behavior of materials in order of micron and sub-micron (Lam, Yang, Chong, Wang, & Tong, 2003; McFarland & Colton, 2005; Stölken & Evans, 1998). The results obtained by these studies show that the classical continuum mechanics theories cannot describe the behavior of such microstructures due to their size dependencies. Thus researchers were stimulated to develop several higher-order continuum theories such as nonlocal theory, strain gradient theory and couple stress theory which could predict size effect by considering material length scale parameters.

In 1960s, the couple stress theory was introduced by Mindlin and Tiersten (1962), Toupin (1962), Mindlin (1964), Koiter (1964) that contains two higher order material length scale parameters in addition to two Lame constants to capture size effect. It should be mentioned that this theory has the lowest material length scale parameters than strain gradient and non-local theory. Using the classical couple stress theory, Anthoine (2000) studied the pure bending of a circular cylinder using two higher order material length scale parameters in addition to the two Lame constants. It should be mentioned that two additional parameters of classical couple stress theory cannot be determined from a single test such as twisting test, bending test or torsion test and just a combination of two tests simultaneously is obligatory to determine these two parameters, so in

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view of the difficulties in determining higher order material length scale parameter for materials by experiments, developing a non-classical theory with one additional parameter was desirable. Therefore, Yang, Chong, Lam, and Tong (2002) introduced a new additional equilibrium equation to solve this problem. The new additional equilibrium equation was equilibrium of moments of couples that was added to the classical equilibrium equations of forces and moments of forces and developed a modified couple stress theory that contains only one higher order material length scale parameters in addition to the two Lame constants. Because of containing only one additional constant, this theory has been used extensively in the recent years by researchers. Park and Gao (2006), Kong, Zhou, Nie, and Wang (2008) are investigated the bending and vibration of Euler–Bernoulli beams respectively. The static bending and free vibration problems of simply supported Timoshenko and Reddy–Levinson beams are studied by Ma, Gao, and Reddy (2008, 2010) respectively. Using modified couple stress theory, Daneshmehr, Mohammad Abadi, and Rajabpoor (2013) studied the thermal effect on static bending, vibration, and buckling analysis of simply supported isotropic beams based on Reddy beam model. Akgöz and Civalek (2011) studied the buckling problem of Euler–Bernoulli micro beams for hinged–hinged and clamped–hinged boundary conditions based on modified couple stress theory and strain gradient elasticity. The authors Mohammad-Abadi and Daneshmehr (2014b) studied the buckling analysis of Euler–Bernoulli, Timoshenko and Reddy micro beams based on modified couple stress theory for several boundary conditions and solved the obtained equations by using the GDQ method. Using the modified couple stress theory, the thermal free vibration and buckling of the size-dependent Timoshenko beams were determined by Ke, Wang, and Wang (2011). In recent years, many researchers have applied modified couple stress theory to analyze size-dependent behavior of microstructures (Akgöz & Civalek, 2014; Asghari, Kahrobaiyan, & Ahmadian, 2010; Dehrouyeh-Semnani & Nikkhah-Bahrami, 2015a, 2015b; Esfahanian, Dehdashti, & Dehrouyeh-Semnani, 2014; Farokhi & Ghayesh, 2015; Khdeir & Reddy, 1994; Şimşek & Reddy, 2013; Wang, Xu, & Ni, 2013; Xia, Wang, & Yin, 2010).

Composite materials have high stiffness-to-weight and strength-to-weight ratios and flexibility in design. Due to these reasons, they exhibit remarkable application potential in tennis racket, aerospace, mechanical specifically automobile engineering structures and civil engineering structures, where weight reduction is critical. In addition, in recent years, substantial use of composite materials in the micro and nano scale in micro electro mechanical system has been noticeable. One can mention microrobots, microsensors, and microactuators as the application of these materials. For instance, Wood, Avadhanula, Menon, and Fearing (2003) investigated the micromechanical flying insect (MFI) made of composite materials in that these materials have a better fracture toughness and fatigue properties of composite materials than semiconductors, and higher stiffness to weight ratios of composite materials than most metals. They reported that the use of micro composite materials has reduced the thorax inertia by a factor of 3 and given a 20% increase in resonant frequency over previous designs. The motion of a stick–slip microrobot in which each leg is a composite beam was studied by Eigoli and Vossoughi (2010). In their study, beam composite was made of piezo-layer and metal base. Ashrafi, Hubert, and Vengallatore (2006) considered the utility of composites as a structural materials for designing the microactuators in micro electro mechanical systems.

Thus, due to the using of composite material in order of micron and sub-micron and also the existence of the micro-scale inhomogeneities and defects such as fibers, impurities and micro-cracks in a laminated composite structure, the theoretical study of micro composite materials is necessary to take into account the effect of material length scale parameters. So some researchers have been motivated to develop micro composite beams based on modified couple stress theory. Chen, Li, and Xu (2011) proposed a new model for composite laminated beam and studied bending analysis of simply supported composite laminated beams with first order shear deformation and solved obtained equation analytically. Based on proposed modified couple stress theory, Wanji, Chen, and Sze (2012) studied the bending of simply supported composite laminated Reddy beam based on modified couple stress theory by defining the new curvature tensor and solved the obtained equations analytically. Chen and Li (2013) investigated the free vibration of simply supported composite laminated Timoshenko beam based on this theory. Roque, Fidalgo, Ferreira, and Reddy (2012) studied the bending of simply supported composite laminated Timoshenko beam and solved the governed equations analytically and numerically. The authors, Mohammad Abadi and Daneshmehr (2014a) developed the buckling analysis of Euler–Bernoulli and Timoshenko beams for hinged–hinged boundary condition and solved the governing equations analytically. In addition, Mohammad-Abadi and Daneshmehr (2015) studied the vibration of three different beam models based on modified couple stress theory for several boundary conditions. Using mentioned theory, the bending of composite laminated Reddy plate was examined by Chen, Xu, and Li (2012).

Micro composite materials may experience high temperature and thermal loads during fabrication and operation. This makes it important to have a good knowledge of the thermal properties of micro composite materials. It is well known that the microbeams' stiffness is reduced by increasing the temperature. Therefore, because of the scarcity of studies on thermal buckling of micro composite laminated beams, in this paper, the thermal effect on the size-dependent buckling analysis of micro composite laminated beams is studied based on modified couple stress theory. The governing equations and boundary conditions are obtained by using the principle of minimum potential energy. To study the effect of shear deformation, different beam models, i.e. composite laminated Euler–Bernoulli beam (CLEBB), Timoshenko beam (CLTB) and Reddy beam (CLRB) models are considered. In addition, the effect of boundary conditions by using the hinged–hinged (h–h), clamped–hinged (c–h) and clamped–clamped (c–c) boundary conditions and the effect of lamination by applying four kinds of cross ply laminate i.e. [0, 0, 0], [0, 90, 0], [90, 0, 90] and [90, 90, 90] are investigated. Using the Fourier series expansions, the governing equations and boundary conditions are solved analytically just for hinged–hinged boundary condition. Also by using the generalized differential quadrature (GDQ) method, the governing equations and boundary conditions are solved numerically for all of the boundary conditions. Comparison between results obtained by GDQ method and analytical solution for