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A note on the unbounded creeping flow past a sphere for Newtonian fluids with pressure-dependent viscosity

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ABSTRACT

We investigate theoretically isothermal, incompressible, creeping Newtonian flows past a sphere, under the assumption that the shear viscosity is pressure-dependent, varying either linearly or exponentially with pressure. In particular, we consider the three-dimensional flow past a freely rotating neutrally buoyant sphere subject to shear at infinity and the axisymmetric flow past a sedimenting sphere. The method of solution is a regular perturbation scheme with the small parameter being the dimensionless coefficient which appears in the expressions for the shear viscosity. Asymptotic solutions for the pressure and the velocity field are found only for the simple shear case, while no analytical solutions could be found for the sedimentation problem. For the former flow, calculation of the streamlines around the sphere reveals that the fore-and-aft symmetry of the streamlines which is observed in the constant viscosity case breaks down. Even more importantly, the region of the closed streamlines around the sphere is absent. Last, it is revealed that the angular velocity of the sphere is not affected by the dependence of the viscosity on the pressure.

1. Introduction

In most isothermal flows of Newtonian liquids, the shear viscosity is commonly assumed to be constant. However, the viscosity of typical liquids begins to increase substantially with pressure when pressures of the order of 1000 atm are reached (Denn, 2008; Rajagopal, Saccomandi, & Vergori, 2012; Renardy, 2003). In certain applications, the effect of the pressure on the viscosity is much larger than that on the mass density, so that compressibility may be neglected but the viscosity pressure dependence needs to be accounted for (Denn, 2008; Goubert, Vermant, Moldenaers, Göttfert, & Ernst, 2001). Hence, the assumption of constant viscosity is valid only at low processing pressures and may introduce error when modeling flows involving high pressures or a large pressure range, e.g. in polymer and food processing, pharmaceutical tablet manufacturing, crude oil and fuel oil pumping, fluid film lubrication, microfluidics, and geophysics (Dealy & Wang, 2013; Le Roux, 2009; Martinez-Boza, Martin-Alfonso, Callegos, & Fernández, 2011; Rajagopal et al., 2012). Due to the growing interest in applications of high pressure chemical and process technologies across a range of engineering fields, flows of fluids with pressure-dependent viscosity as well as techniques for measuring the pressure dependence of the viscosity and the viscosity at high pressure have received increased attention recently (Goubert et al., 2001; Park, Lim, Laun, & Dealy, 2008; Schaschke, 2010). The pressure dependence of viscosity, however, is not only of industrial but also of great fundamental importance.

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Málek and Rajagopal (2007) reviewed different empirical equations proposed in the literature in order to describe experimental observations on the pressure-dependence of the viscosity. Barus (1893) proposed the following formula for the viscosity, η^*:

$$\eta^* = \eta_0^* \exp\left[\delta^*(p^* - p_0^*)\right] \tag{1}$$

where δ^* is the pressure-dependence coefficient, assumed to be constant, and η_0^* is the viscosity at the reference pressure p_0^*. It should be noted that throughout the text a superscript * denotes a dimensional quantity. Barus (1891) employed the following linear expression:

$$\eta^* = \eta_0^*\left[1 + \delta^*(p^* - p_0^*)\right] \tag{2}$$

which is equivalent to Eq. (1) for small values of δ^* and/or small pressure differences. As noted by Schaschke (2010), simple models like the above may be used only for simple and small molecules and not for long chain molecules, such as polymers and oil mixtures. Other formulae proposed in the literature, which better fit experimental results for complex fluids, can be found in Málek and Rajagopal (2007).

The pressure-dependence of the viscosity in lubrication (Szeri, 1998), viscometric and other flows has been analyzed mathematically by various investigators (Hron, Málek, & Rajagopal, 2001; Lanzendörfer & Stebel, 2011; Málek & Rajagopal, 2007; Marušić & Pažanin, 2013; Renardy, 2003). Hron et al. (2001) studied various unidirectional and two-dimensional flows of simple fluids with pressure-dependent viscosities and showed that unidirectional flows corresponding to Couette or Poiseuille flow are possible only in special forms of the viscosity. Kalogirou, Poyiadji, and Georgiou (2011) compiled analytical solutions for internal, Poiseuille-type, steady flows. More specifically, these authors studied the unidirectional plane, axisymmetric and annular Poiseuille flows of a Newtonian liquid assuming that the viscosity obeys Eq. (2). Pruša, Srinivasan, and Rajagopal (2012) have recently investigated the role of pressure-dependent viscosity in measurements with falling cylinder viscometers and showed that the error introduced by the application of the classical constant-viscosity formula can be significant for some fluids. They also proposed a heuristic correction to that formula.

So far, however, investigation of external flows with pressure-dependent viscosity are almost absent from the literature. As far as we are aware, no analytical solutions exists, and only some limiting numerical results for the unbounded axisymmetric flow past a sedimenting sphere in a power-law ambient fluid have been presented by Chung and Vaidya (2010). Although in external inertialess flows, one does not expect very large variations of the pressure due to flow, the non-linearity of the governing equations may be adequate to predict unexpected and new flow phenomena.

Of particular interest to the present work is the flow around a sphere, i.e. the simplest flow relevant to falling body type viscometry. More specifically, we have chosen to study the influence of a pressure-dependent shear viscosity on the steady, creeping, incompressible flow of a Newtonian liquid of mass density ρ_f^* past a rigid sphere of radius R^* and mass density ρ_s^*. Two different cases are investigated. In the first case, the mass densities of the fluid and the sphere are assumed to be equal, i.e. the sphere is neutrally buoyant, and shear is applied far from the sphere. In the second case, $\rho_s^* > \rho_f^*$ but no shear is applied, and thus the sphere sediments with a constant terminal velocity.

For a neutrally buoyant sphere under the influence of simple shear flow imposed far from the sphere, it is known that non-linear effects, such as inertia or viscoelasticity, break the fore-and-aft symmetric configuration of the streamlines around the sphere, in the plane which shear is applied. Indeed, Lin, Peery, and Schowalter (1970) and Subramanian and Koch (2007), among others, have shown that the inclusion of the inertia terms into the governing equations destroys the symmetry of the orbits of the fluid elements around the sphere. This may have important consequences for the heat transfer around the spherical particle, as well for the bulk properties of suspensions of particulates (Subramanian & Koch, 2007). D'Avino et al. (2008) and Housiadas and Tanner (2011) considered the case where the matrix fluid is a viscoelastic fluid and showed that the fore-and-aft symmetry of the streamlines also then breaks down. In the present work, it is shown that an alternative cause of the destruction of the streamline symmetry around a spherical particle is the non-linearity introduced by the pressure dependence of the viscosity of the matrix fluid.

Recently, is has been demonstrated that for internal, fully developed, laminar flows in straight channels and circular tubes the perturbation solution up to fourth order in δ (for the definition of the dimensionless coefficient δ see the subsequent section) is an excellent approximation of the full, analytical solution (Poyiadji, Housiadas, Kaouri, & Georgiou, 2015). For the problems under consideration, and for typical flow conditions, the dimensionless pressure-viscosity coefficient is a small number, i.e. $\delta \ll 1$. Since the full, non-linear, governing equations cannot be solved analytically, we employed a regular perturbation scheme with the small number being the δ coefficient.

The rest of the paper is organized as follows. In Section 2, the assumptions, governing equations and boundary conditions are presented in dimensionless form. In Section 3, the solution procedure and the analytical solution up to first order is presented for the simple shear case. In the same section, the streamlines around the sphere are calculated and discussed. The main conclusions are summarized in Section 4. Finally, in Appendix A, it is shown that no analytical solution can be found with the proposed method for the two-dimensional flow past a sedimenting sphere.

2. Problem definition

The steady, creeping, isothermal flow around a sphere is considered. The ambient fluid is assumed to be Newtonian with constant mass density ρ_f^*, and a variable shear viscosity η^*, given by either one of Eqs. (1) and (2). A fixed spherical