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Employing of the discrete Fourier transform for evaluation of crack-tip field in periodic materials

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ABSTRACT

An approach for numerical evaluation of a self-similar crack-tip field for a long (semi-infinite) crack embedded in a material with periodic microstructure is suggested. The conditions at the boundaries of a rectangular domain around the tip are formulated by the use of *K*-field for the homogeneous material possessing effective elastic properties and then the finite discrete Fourier transform is applied. This allows to replace standard analysis of a large periodic domain with many cells by the analysis of a single repetitive cell in the transform space which can be carried out by any numerical method. Consequently, the volume of calculations in comparison with the standard approach is reduced and the problem of a macrocrack embedded in a material with fine microstructure can be addressed without simplifying assumptions. The accuracy of the proposed approach is verified by a comparison with the analytical solution for a crack embedded in a homogeneous plane.

Application of the suggested method is given for a crack in a two-dimensional periodically voided material with triangular isotropic layout. The cell problem is resolved by the finite element method. The fracture toughness of the material in the framework of stress criterion for crack propagation is determined and its dependence upon the material relative density is investigated. A comparison of the fracture toughnesses of the solid and voided materials has shown for which parameter combinations voided ones will provide better crack resistance.

1. Introduction

Investigation of new micro-architectured materials with improved fracture properties is one of the current research challenges in material science (Fleck, Deshpande, & Ashby, 2010). One of the basic problems in this topic is the analysis of the crack tip field in materials with a periodic microstructure. This problem, as a rule, is a computationally expensive task due to the different scales involved in the analysis. The reason for this is that the basic stipulation for the existence of the self-similar stress field, which is of special interest, is that the crack length a significantly exceeds the characteristic size of the periodic module l

$$a \gg l. \tag{1}$$

The conventional approach (*K*-field approach) to the near-tip field derivation is based on the matching of the elastic field in the near-tip non-homogeneous periodic domain with the *K*-field of semi-infinite crack in the homogeneous elastic material

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possessing effective elastic properties (e.g., Jha & Charalambides, 1998; Leguillon & Piat, 2008). The size of the domain must be large enough to meet the mentioned stipulation and, consequently, the domain includes a large number of repetitive modules. Therefore its fine meshing which is required for the adequate microstructure modeling may lead to a huge number of degrees of freedom.

The advocated technique allows to obtain without difficulty an accurate stress distribution in an arbitrarily large periodic domain surrounding the crack tip. It is applicable for any multiphase periodic composites. In the present study two-dimensional voided materials will be considered when one phase vanishes. For these materials the crack tip can be associated with a void region, the stress field is non-singular and the fracture toughness can be evaluated in terms of tensile strength of the parent solid material σ_f.

In the case of cellular materials with relatively large voids, beam lattice approximation is applicable for which nodes displacements and rotations completely define the stress state. Consequently, the number of degrees of freedom in a repetitive module is small and consideration of a sufficiently large domain for the fracture toughness evaluation based on the K-field approach is not troublesome (Fleck & Qiu, 2007; Thiyagasundaram, Wang, Sankar, & Arakere, 2011). For denser voided materials when the beam model becomes invalid the direct modeling of a large domain is impractical due to the large number of degrees of freedom and, therefore, the K-field approach was not applied. Several results for these type of materials were obtained by the use of the representative cell method based on the discrete Fourier transform (Lipperman, Ryvkin, & Fuchs, 2008; Ryvkin & Aboudi, 2011). This method reduces the volume of calculations by replacing the problem for a rectangular domain with a crack by several problems for a representative periodic cell. The number of the representative cell problems is equal to the total number of cells in the domain. On the other hand, a certain disadvantage in the employed framework for the representative cell method is that, contrary to the K-field approach, a finite length crack embedded in the middle of the domain is to be considered and, consequently, the following relation between the problem length scales is to be fulfilled

$$s \gg a \gg l, \tag{2}$$

where s is the overall size of the domain. In the present work a new method combining the positive features of the K-field and the representative cell approaches is suggested.

In the next section the conventional problem for derivation of the stress field in the vicinity of a semi-infinite crack in voided material is presented. In Section 3 this problem is reformulated by the use of jump functions which allowed to apply the representative cell method for its solution as it is shown in Section 4. An application of the developed methodology for the study of fracture behavior of a plate with double periodic array of circular voids is presented in Section 5 and in the final section several conclusions are drawn.

2. Semi-infinite crack in a periodically voided material (Problem A)

The K-field approach to the fracture toughness problem, which is in common use in the analysis of cellular materials may be formulated as following in the considered case of voided material. Consider a semi-infinite crack $-\infty < X_1 < 0,\ X_2 = 0$ embedded in two-dimensional periodic voided material. The boundary problem is formulated for a rectangular domain $-L_r \leqslant X_r \leqslant L_r,\ r = 1, 2$ around the crack tip (Fig. 1a). The dashed lines in the figure should be disregarded at this stage. The displacement vector $\mathbf{u}$ and stress tensor $\boldsymbol{\sigma}$ satisfy the field equations of the plane problem of elasticity defined by the differential operator $\mathcal{L}$

$$\mathcal{L}[\mathbf{u}, \boldsymbol{\sigma}] = 0, \tag{3}$$

the tractions $\mathbf{t}_v$ at the voids boundaries as well as at the crack faces $\mathbf{t}_{cr}$ vanish

$$\mathbf{t}_v = 0, \tag{4}$$

$$\mathbf{t}_{cr} = 0, \tag{5}$$

and at the outer boundaries of the rectangle the tractions defined by the K-field eigensolution for the stresses in homogeneous material possessing the effective elastic properties are applied

$$\mathbf{t}(\pm L_1, X_2) = \mathbf{t}^K(\pm L_1, X_2), \tag{6}$$

$$\mathbf{t}(X_1, \pm L_2) = \mathbf{t}^K(X_1, \pm L_2). \tag{7}$$

In the case when the rectangle boundaries cross the voids, as shown in the figure, a corresponding averaging procedure is to be applied. The expressions for the components of the K-field for cracks in anisotropic material can be found, for example, in Sih, Paris, and Irwin (1965).

Alternatively, the conditions at the rectangle boundaries can be formulated in terms of displacements $\mathbf{u}^K = \{u_1^K, u_2^K\}$ corresponding to the displacements K-field in the homogenized material

$$\mathbf{u}(\pm L_1, X_2) = \mathbf{u}^K(\pm L_1, X_2), \tag{8}$$

$$\mathbf{u}(X_1, \pm L_2) = \mathbf{u}^K(X_1, \pm L_2). \tag{9}$$