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Exact elasto-plastic analysis of rotating thick-walled cylindrical pressure vessels made of functionally graded materials

Mohammad Zamani Nejad *, Parisa Fatehi

Mechanical Engineering Department, Yasouj University, P.O. Box 75914-353, Yasouj, Iran

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ABSTRACT

The objective of this study is to obtain the exact elasto-plastic deformations and stresses of rotating thick-walled cylindrical pressure vessels made of functionally graded materials (FGMs) under plane strain condition. The plastic stresses and deformations are obtained, using Tresca's yield condition, and its flow rule under the assumption of perfectly plastic material behavior. The material properties are assumed to vary according to power law functions, and the Poisson's ratio is assumed constant. To the best of the authors' knowledge, in previous studies in which exact elasto-plastic behavior of rotating FGM thick-walled cylindrical pressure vessels was investigated, variation of material density was ignored, while density is not constant in this type of structures. In the present work, rotation, internal and external pressure and variation of material properties are considered simultaneously.

1. Introduction

Functionally graded materials (FGMs) are heterogeneous composite materials whose properties change smoothly and continuously along desired dimension(s). This continuously varying composition eliminates interface problems, and thus, the stress distributions are smooth (Fatehi & Nejad, 2014; Zenkour, 2013). A number of papers considering various aspects of FGM have been published in recent years (Ghannad, Rahimi, & Nejad, 2013; Kahrobaiyan, Rahaeifard, Tajalli, & Ahmadian, 2012; Nejad, Rastgoo, & Hadi, 2014; Nejad & Kashkoli, 2014; Simsek & Reddy, 2013; Xue & Pan, 2013).

Elasto-plastic analysis of FGM thick-walled cylindrical pressure vessels has been intensively investigated in the literature. Nadai (1950) treated the elastic deformation of rotating perfectly plastic cylinders in 1950. He assumed that only one plastic region occurs at the center of the cylinder at elastic limit rotation speed. He gained stresses for this region; however, he did not check the kinematical continuity at the border of the elastic and plastic regions. Hodge and Balaban (1962) attempted to solve the problem encountered by Nadai under more general conditions, assuming the same plastic regime as in Nadai (1950). Lenard and Haddow (1972) achieved instability speed for rotating solid cylinders. They also took for granted Nadai's misconceptions for plastic flow. Stresses and radial displacement in rotating linearly hardening hollow tube with fixed ends were obtained by Gamer and Lance (1983). Gamer and Sayir (1984) corrected the common mistake in the above-mentioned studies in 1984. Among other things, they found that when yielding starts at the center of the rotating solid shaft, another plastic region in a side regime appears simultaneously. Mack (1991) studied the problem of rotating elasto-plastic hollow tubes with free ends in 1991. In another study, Mack (1992) estimated the unloading and the secondary flow in a rotating

* Corresponding author. Fax: +98 7422331711.
E-mail addresses: m.zamani.n@gmail.com, m_zamani@yu.ac.ir (M.Z. Nejad).

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elastic–plastic hollow cylinder. Orcan (1994), using Tresca's yield condition and its flow rule under the assumption of perfectly plastic material behavior achieved analytically thermally induced deformations of a cylinder with free ends. Later Gamer, Mack, and Varga (1997) obtained the distribution of stress, displacement and plastic strain of an elastic–perfectly plastic rotating solid shaft with fixed ends. Lindner and Mack (1998) obtained the residual stresses in a rotating elastic–perfectly plastic solid shaft with fixed ends. Eraslan (2003) determined the elastic–plastic deformations of a rotating linearly strain hardening solid shaft. Eraslan (2004) investigated the stress distributions in rotating elastic–plastic solid and hollow shafts, using von Mises'yield criterion. He determined stresses, displacement and plastic strains for nonlinearly hardening elastic–plastic solid and hollow shafts rotating at different speeds. Eraslan and Akis (2005) achieved elasto-plastic response of a long functionally graded pressurized tube based on Tresca's yield criterion, assuming ideally plastic material behavior. They assumed that all properties are constant but the modulus of elasticity varies radially. Akis and Eraslan (2006) obtained an analytical solution for the stress response and onset of yield of rotating FGM hollow shafts, using generalized plane strain assumption. The modulus of elasticity and the uniaxial yield limit of the shaft material varied according to power-law function, but they again considered the material density as a constant. Eraslan and Akis (2006a) obtained analytical solutions for FG rotating solid shaft and rotating solid disk, using plane strain and plane stress conditions, respectively. Following that, Eraslan and Akis (2006b) assumed the modulus of elasticity and yield limit to vary radially in general parabolic forms and investigated an analytical solution of a plane strain functionally graded pressurized tube. In still another study, Eraslan and Akis (2006c) presented elasto-plastic response of rotating functionally graded hollow shafts with fixed ends but they considered only one plastic region in their calculations. It is worth mentioning that they considered the material density as a constant in FG materials. The modulus of elasticity of the material varied in both exponential and parabolic forms but they considered the material density as a constant in FG materials. Akis and Eraslan (2007) defined the elastic, partially plastic and fully plastic deformation behavior of rotating FGM hollow shafts with fixed ends. They assumed all properties to be constant but the modulus of elasticity to vary radially. They found that the plastic deformation may start at the inner or outer surface or even simultaneously at both surfaces. Eraslan and Arslan (2007) presented analytical solutions to rotating partially plastic graded hollow shafts. They assumed all properties to be constant but the modulus of elasticity to vary radially. They showed that rotation speed of the shaft is almost independent of the variation in the modulus of elasticity of the material. Eraslan, Arslan, and Mack (2007) achieved the distribution of stress, strain and displacement in a linearly strain hardening rotating hollow shaft subjected to a positive temperature gradient. They found that a temperature difference between the outer and the inner surface decreases the elastic limit angular speed but it may increase the fully plastic angular speed. Gao (2007) obtained an elastic–plastic solution for an internally pressurized thick-walled cylinder under the assumption of plane strain condition. Material was assumed as an elastic linear-hardening plastic material. Dixon and Perez (2008) derived an analytical solution for open-ended thick walled cylinders based on the Von Mises yield criterion for the collapse pressure. Darijani, Kargarnovin, and Naghdabadi (2009) presented an exact elasto-plastic solution for a thick-walled cylindrical vessel made of elastic linear-hardening material, considering the Bauschinger effect and the Tresca yield criterion. Sharma, Sahni, and Kumar (2009) investigated elastic–plastic stresses of a transversely isotropic thick-walled rotating cylinder subjected to internal pressure, using Seth's transition theory. They found that circular cylinders made of transversely isotropic material is safer to design as compared with cylinders made of isotropic material. Ozturk and Gulgec (2011) obtained elastic–plastic deformation of FGM solid cylinders with fixed ends, under uniform internal heat generation, using Tresca's yield criterion and its associated flow rule. The materials of cylinders were supposed to be elastic–perfectly plastic materials. Pankaj (2011) presented the problem of elastic–plastic stresses of thick walled rotating cylinders by finite deformation under steady-state temperature. They used the concept of *generalized strain measure* and assumed material density as constant. Parvizi, Naghdabadi, and Arghavani (2011) investigated an analytical elastic–plastic solution for FGM thick-walled cylinders under internal pressure and thermal loading. It was shown that there is a point in the cylinder where the circumferential stress changes from compressive to tensile under the temperature gradient loading. Rajabi and Darvizeh (2012) solved elasto-plastic deformation of circular cylindrical shells under internal electromagnetic forces, assuming elastic–plastic material and non-linear plastic behavior. They used first-order shear deformation theory (FSDT), deformation theory of plasticity and von Mises yield criteria to derive the equations. Sharma and Yadav (2013) presented the thermal elastic–plastic stresses of a rotating FG stainless steel composite cylinder under internal and external pressure. They used general nonlinear strain hardening law, von Mises' yield criterion and the finite difference method to derive the equations. Fatehi and Nejad (2014) presented elastic solution of rotating thick hollow cylindrical shells made of functionally graded materials, using the infinitesimal theory of elasticity and also determined the location of yield, using both Tresca's and von Mises yield criteria. In that work, rotation, internal and external pressure and variation of material properties were considered simultaneously. They considered the material density, like other material properties, to vary according to power law functions, and showed that variation of density has significant impacts on the onset of yield and the stress distributions. They also investigated the effects of variation of Poisson's ratio and non-homogenous parameters on the onset of yield.

The aim of the present study is to determine analytical solutions to predict the partially plastic stress responses of FGM thick-walled cylindrical pressure vessels with fixed ends subjected to uniform internal and external pressure. To fulfill this, material density, like other properties, is assumed to vary nonlinearly. In addition, Poisson's ratio, ν, is assumed constant. First, the elastic analysis of a rotating FGM thick cylindrical pressure vessel is presented. Then, the stresses and deformations of three different plastic regions are presented, using Tresca's yield condition, and its flow rule under the assumption of perfectly plastic material behavior.